

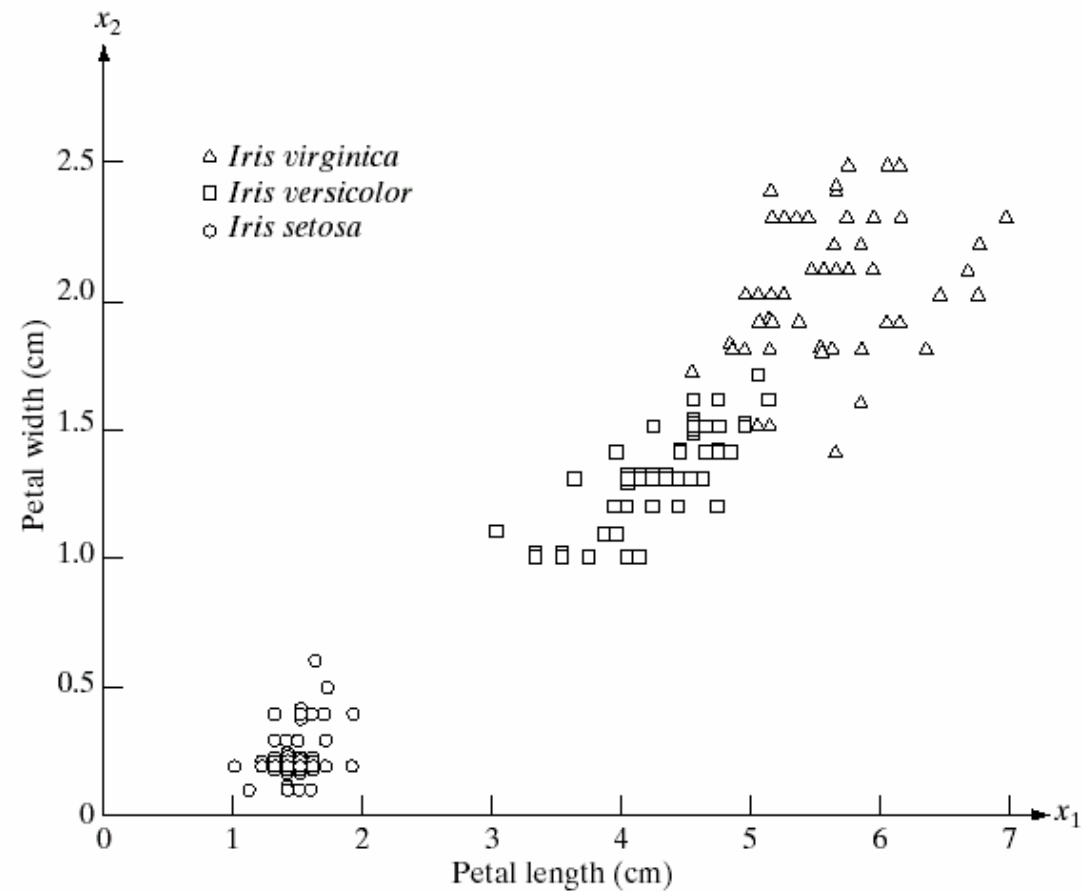


## Chapter 12

### Object Recognition

**FIGURE 12.1**

Three types of iris flowers described by two measurements.





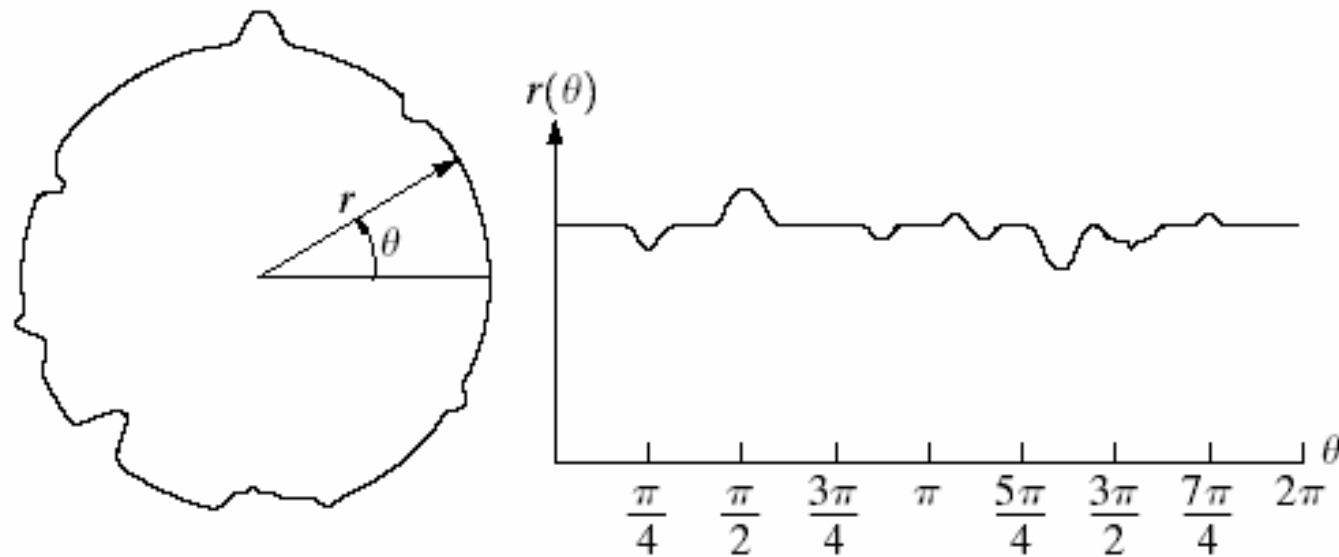
# Patterns and Pattern Classes

- A *pattern* is an *arrangement of descriptors*, such as those discussed in Chapter 11.
- The name *feature* is used often in the pattern recognition literature to denote a descriptor.
- A *pattern class* is a family of patterns that share some common properties.



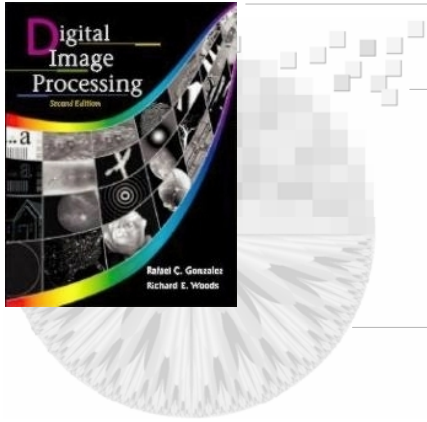
## Chapter 12

# Object Recognition



a b

**FIGURE 12.2** A noisy object and its corresponding signature.

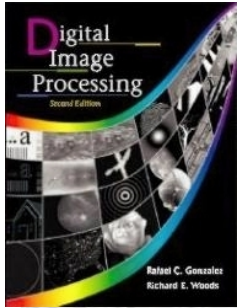


# Chapter 12

## Object Recognition

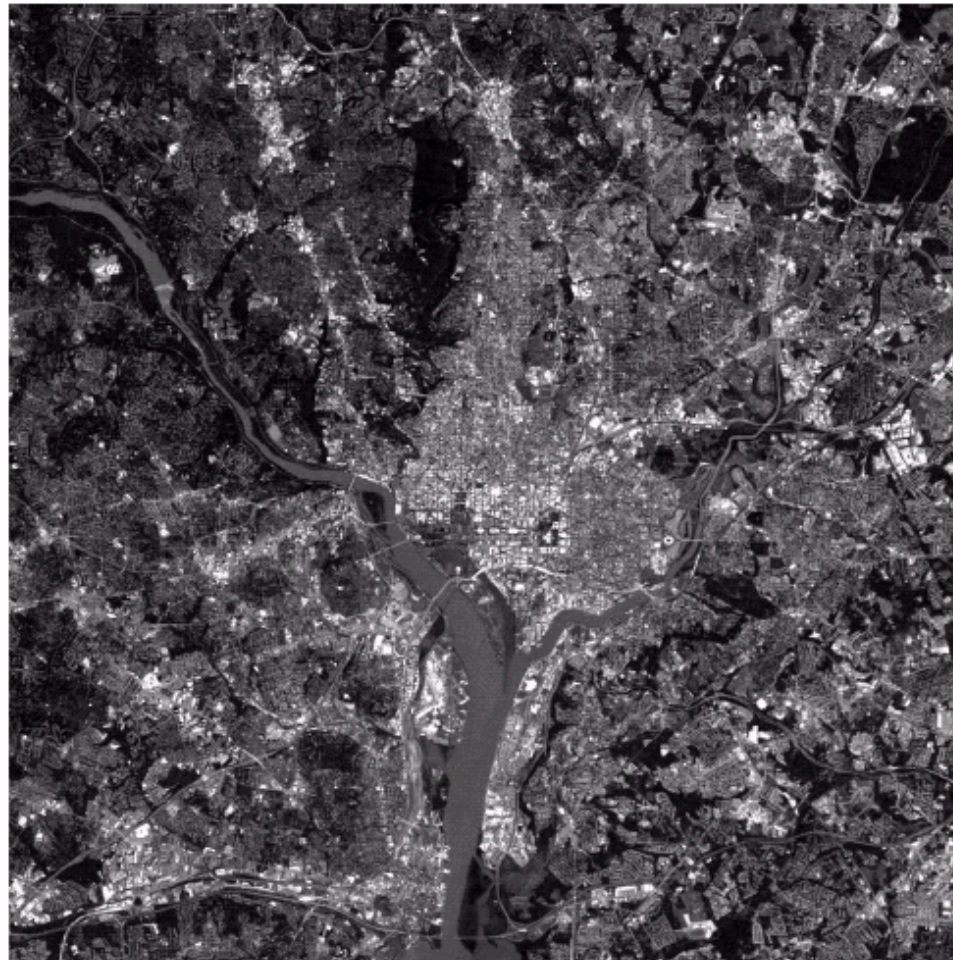


**FIGURE 12.3** (a) Staircase structure. (b) Structure coded in terms of the primitives  $a$  and  $b$  to yield the string description  $\dots ababab \dots$ .

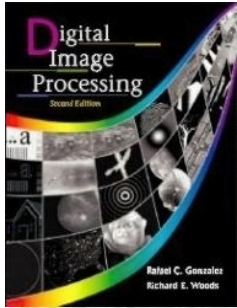


## Chapter 12

# Object Recognition

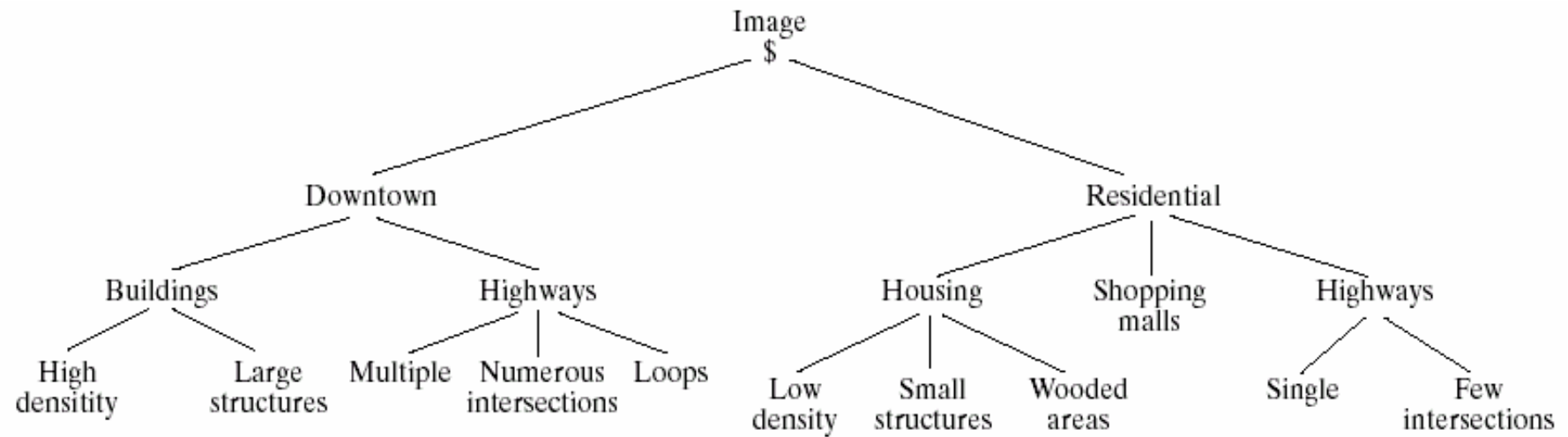


**FIGURE 12.4**  
Satellite image of  
a heavily built  
downtown area  
(Washington,  
D.C.) and  
surrounding  
residential areas.  
(Courtesy of  
NASA.)



## Chapter 12

# Object Recognition



**FIGURE 12.5** A tree description of the image in Fig. 12.4.





# Recognition Based on Decision-Theoretic Methods

Let  $x = (x_1, x_2, \dots, x_n)^T$  for  $W$  pattern classes  $\omega_1, \omega_2, \dots, \omega_W$   
 $d_i(x) > d_j(x) \quad j = 1, 2, \dots, W; j \neq i$

- In other words, an unknown pattern  $\mathbf{x}$  is said to belong to the  $i$ th pattern class if, upon substitution of  $\mathbf{x}$  into all decision functions,  $d_i(x)$  yields the largest numerical value.



# Matching

## Minimum distance classifier

- Suppose that we define the prototype of each pattern class to be the mean vector of the patterns of that class:  $m_j = \frac{1}{N_j} \sum_{x \in \omega_j} x_j \quad j = 1, 2, \dots, W$
- We then assign  $\mathbf{x}$  to class  $\omega_i$  if  $D_i(\mathbf{x})$  is the smallest distance.  $D_j(x) = \|x - m_j\|$





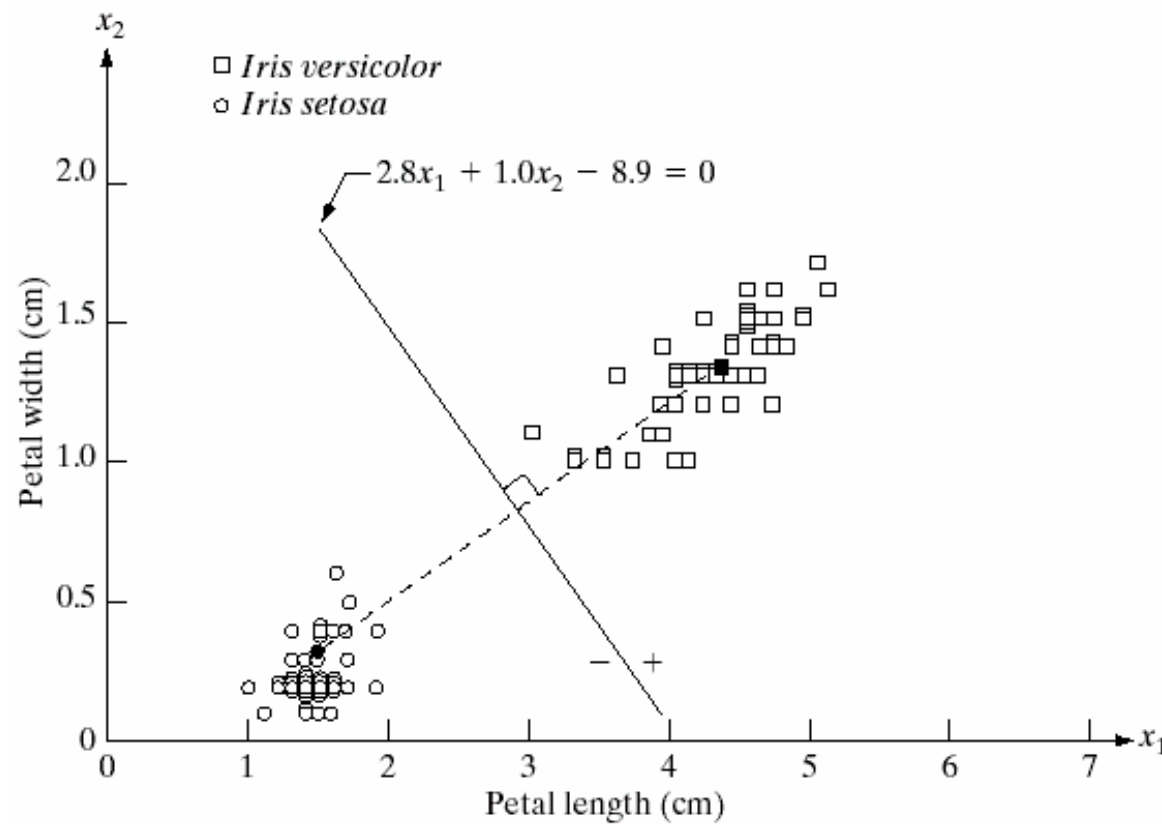
# Minimum distance classifier

- It is not difficult to show (Problem 12.2) that selecting the smallest distance is equivalent to evaluating the functions  $d_j(x) = x^T m_j - \frac{1}{2} m_j^T m_j \quad j = 1, 2, \dots, W$
- assign  $\mathbf{x}$  to class  $\omega_i$  if  $d_i(\mathbf{x})$  is the largest numerical value.
- This formulation agrees with the concept of a decision function, as defined in Eq. (12.2-1).



## Chapter 12

### Object Recognition

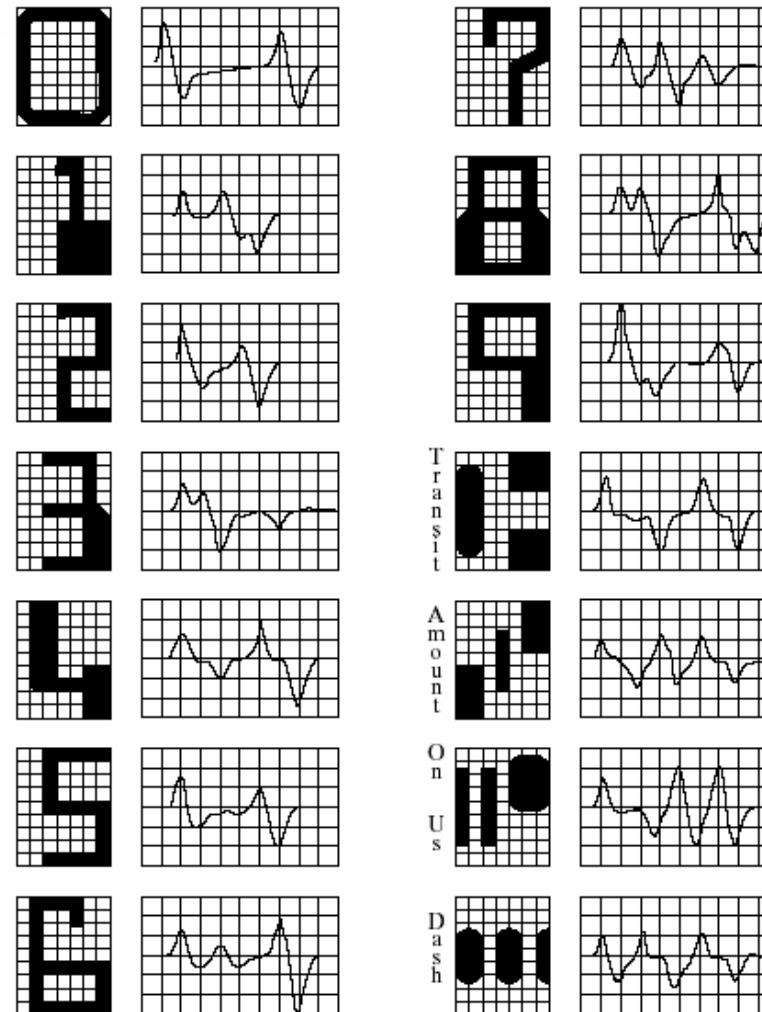


**FIGURE 12.6**  
Decision boundary of minimum distance classifier for the classes of *Iris versicolor* and *Iris setosa*. The dark dot and square are the means.



## Chapter 12

# Object Recognition



**FIGURE 12.7**  
American  
Bankers  
Association  
E-13B font  
character set and  
corresponding  
waveforms.

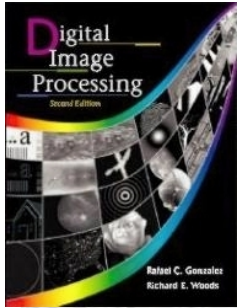


# Matching by correlation

$$c(x, y) = \sum_s \sum_t f(s, t) w(x + s, y + t)$$

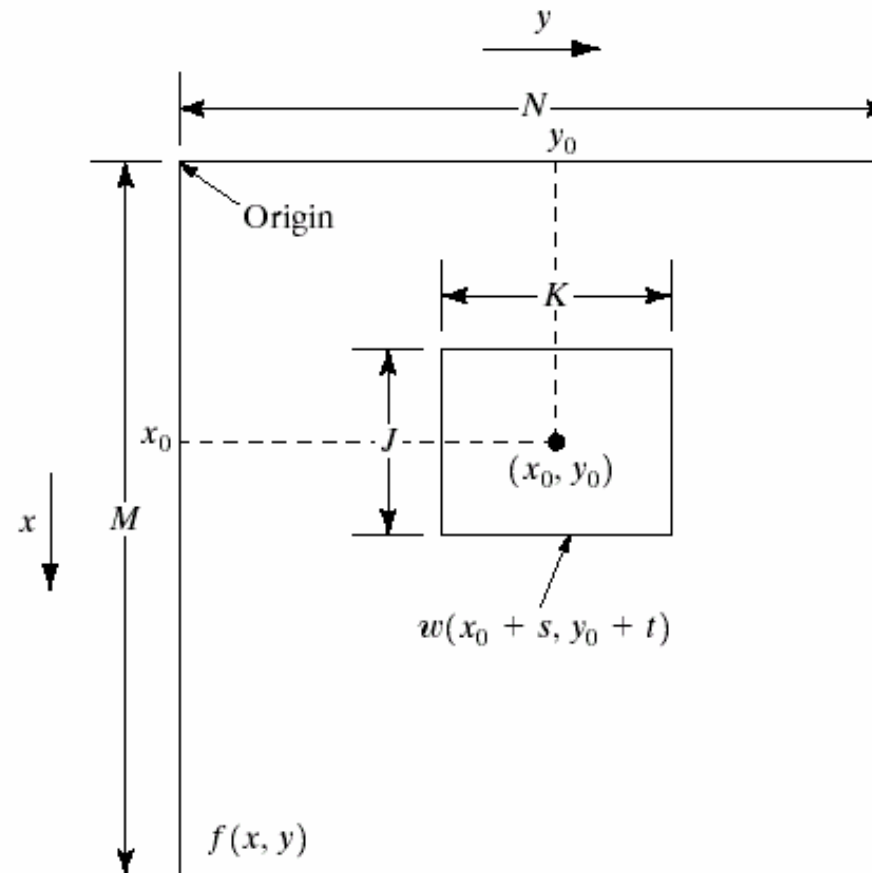
- *correlation coefficient*, which is defined as

$$\gamma(x, y) = \frac{\sum_s \sum_t [f(s, t) - \bar{f}(s, t)] [w(x + s, y + t) - \bar{w}]}{\left\{ \sum_s \sum_t [f(s, t) - \bar{f}(s, t)]^2 \sum_s \sum_t [w(x + s, y + t) - \bar{w}]^2 \right\}^{\frac{1}{2}}}$$

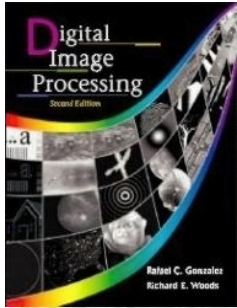


## Chapter 12

# Object Recognition

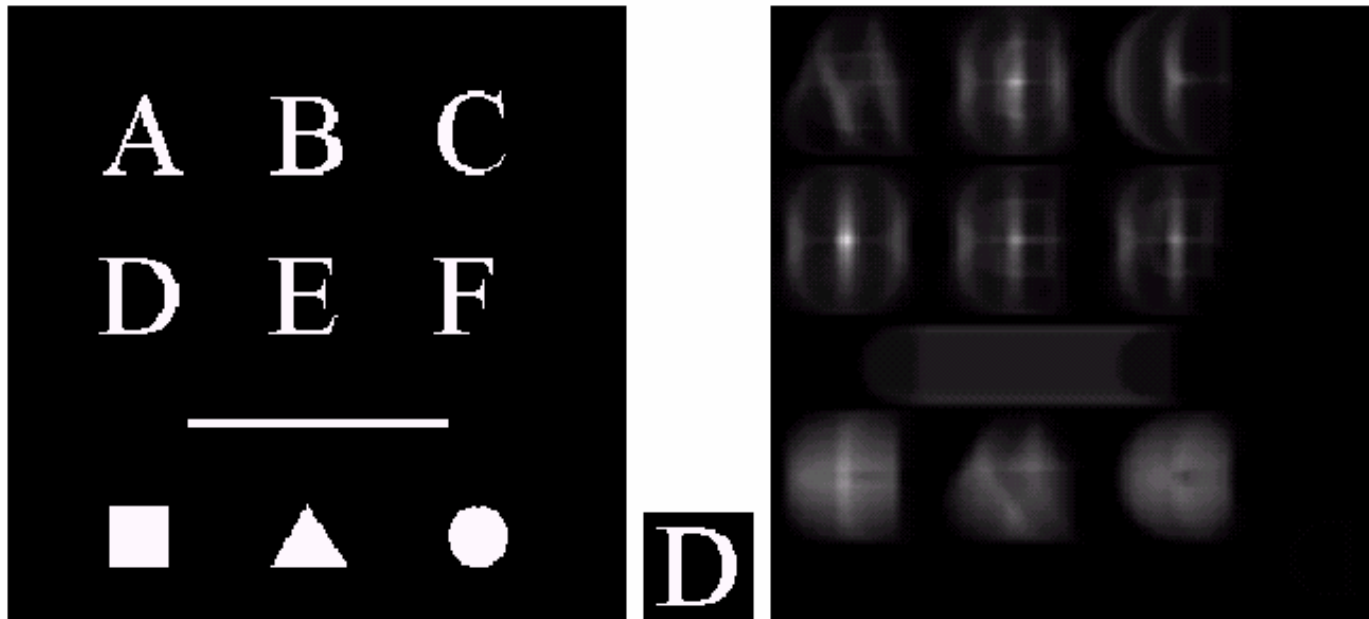


**FIGURE 12.8** Arrangement for obtaining the correlation of  $f$  and  $w$  at point  $(x_0, y_0)$ .



## Chapter 12

# Object Recognition

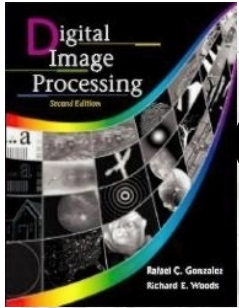


a b c

**FIGURE 12.9**

(a) Image.  
(b) Subimage.  
(c) Correlation coefficient of (a) and (b). Note that the highest (brighter) point in (c) occurs when subimage (b) is coincident with the letter “D” in (a).





# Optimum Statistical Classifiers

- $r_j(x) = \sum_{k=1}^W L_{kj} p(\omega_k / x)$  the equation often called the *conditional average risk* or *loss* in decision-theory terminology.
- The classifier that minimizes the total average loss is called the *Bayes classifier*.
- Thus the Bayes classifier assigns an unknown pattern  $\mathbf{x}$  to class  $\omega_i$  if  $r_i(x) < r_j(x)$  for  $j = 1, 2, \dots, W; j \neq i$   
in other words,  $\mathbf{x}$  is assigned to class  $\omega_i$  if

$$\sum_{k=1}^W L_{ki} p(\omega_k / x) P(\omega_k) < \sum_{q=1}^W L_{qj} p(x / \omega_q) P(\omega_q)$$



# Bayes classifier for Gaussian pattern classes

$$d_j(x) = p(x / \omega_j) P(\omega_j) = \frac{1}{\sqrt{2\pi}\sigma_j} e^{-\frac{(x-m_j)^2}{2\sigma_j^2}} P(\omega_j) \quad j = 1, 2$$

$$p(x / \omega_j) = \frac{1}{(2\pi)^{n/2} |C_j|^{n/2}} e^{-\frac{1}{2}(x-m_j)^T C_j^{-1} (x-m_j)} \quad m_j = E_j \{x\}$$

$$C_j = E_j \{ (x - m_j)(x - m_j)^T \} \quad C_j = \frac{1}{N_j} \sum_{x \in \omega_j} xx^T - m_j m_j^T$$

Bayes decision function for class  $\omega_j$  is  $d_j(x) = p(x / \omega_j) P(\omega_j)$

$$d_j(x) = \ln P(\omega_j) + x^T C^{-1} m_j - \frac{1}{2} m_j^T C^{-1} m_j$$

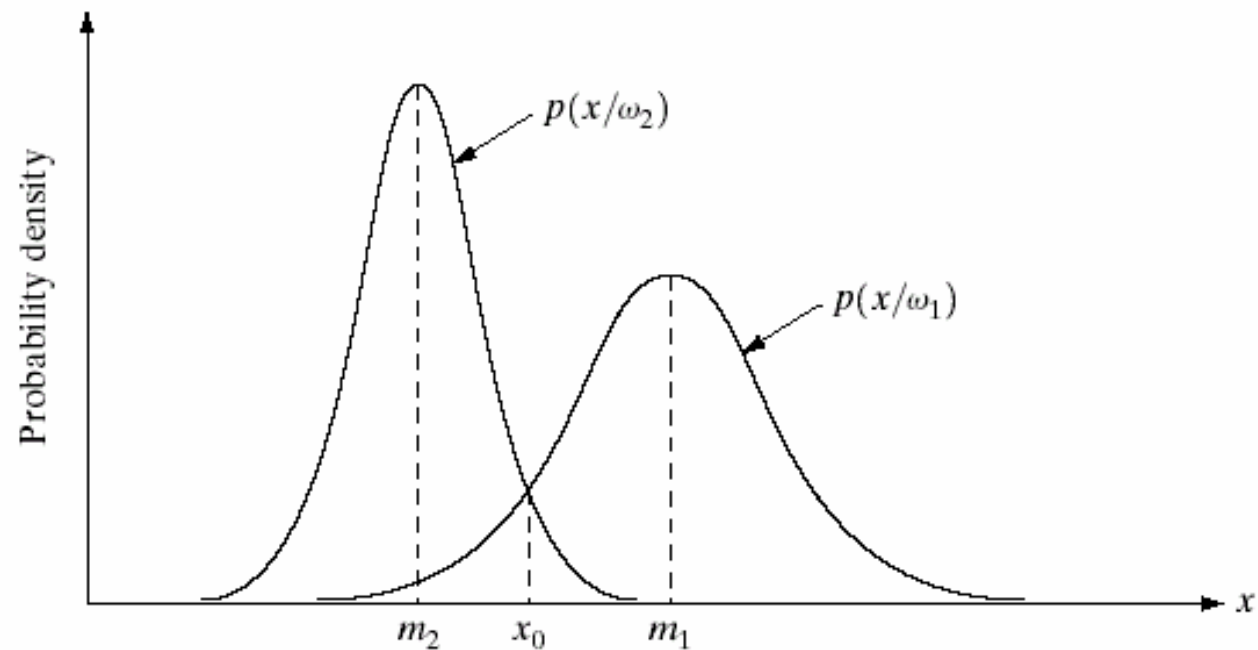


## Chapter 12

# Object Recognition

**FIGURE 12.10**

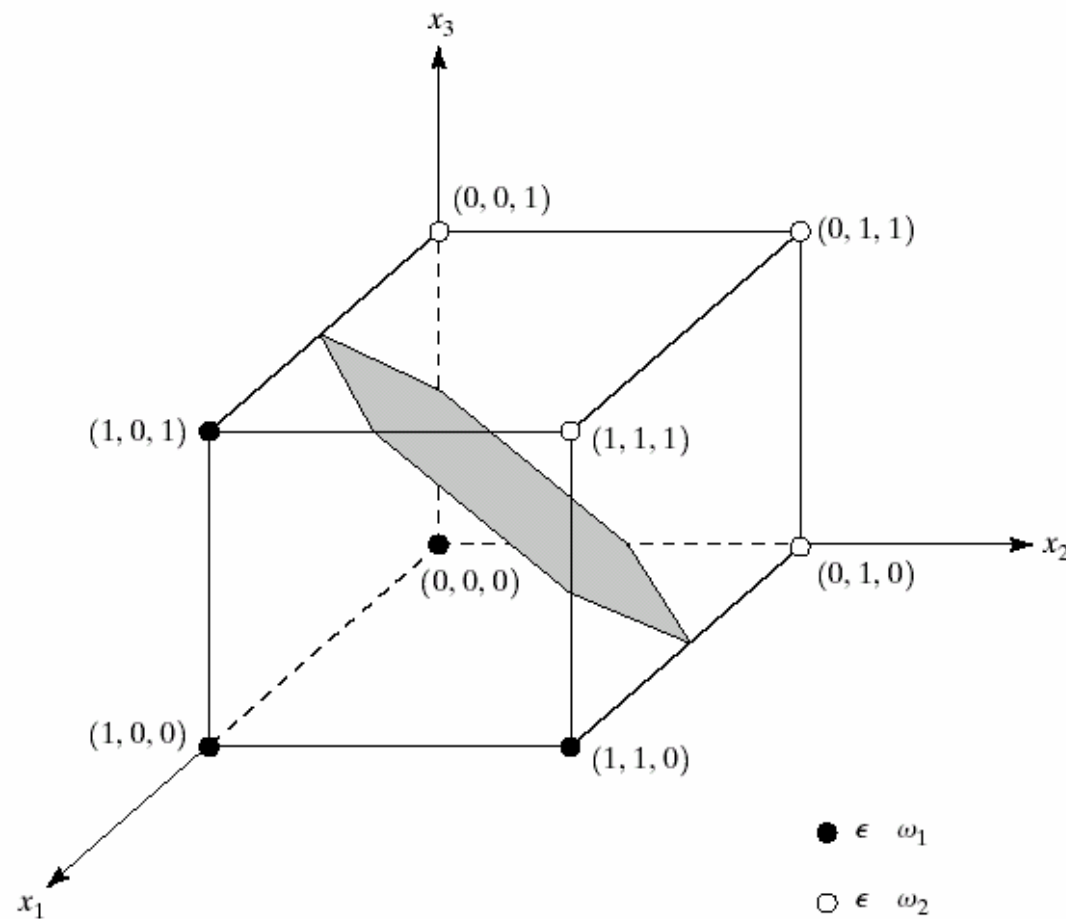
Probability density functions for two 1-D pattern classes. The point  $x_0$  shown is the decision boundary if the two classes are equally likely to occur.



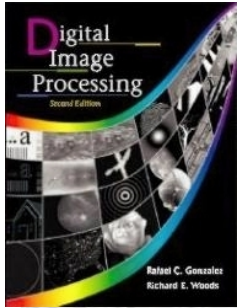


## Chapter 12

# Object Recognition



**FIGURE 12.11**  
Two simple  
pattern classes  
and their Bayes  
decision boundary  
(shown shaded).

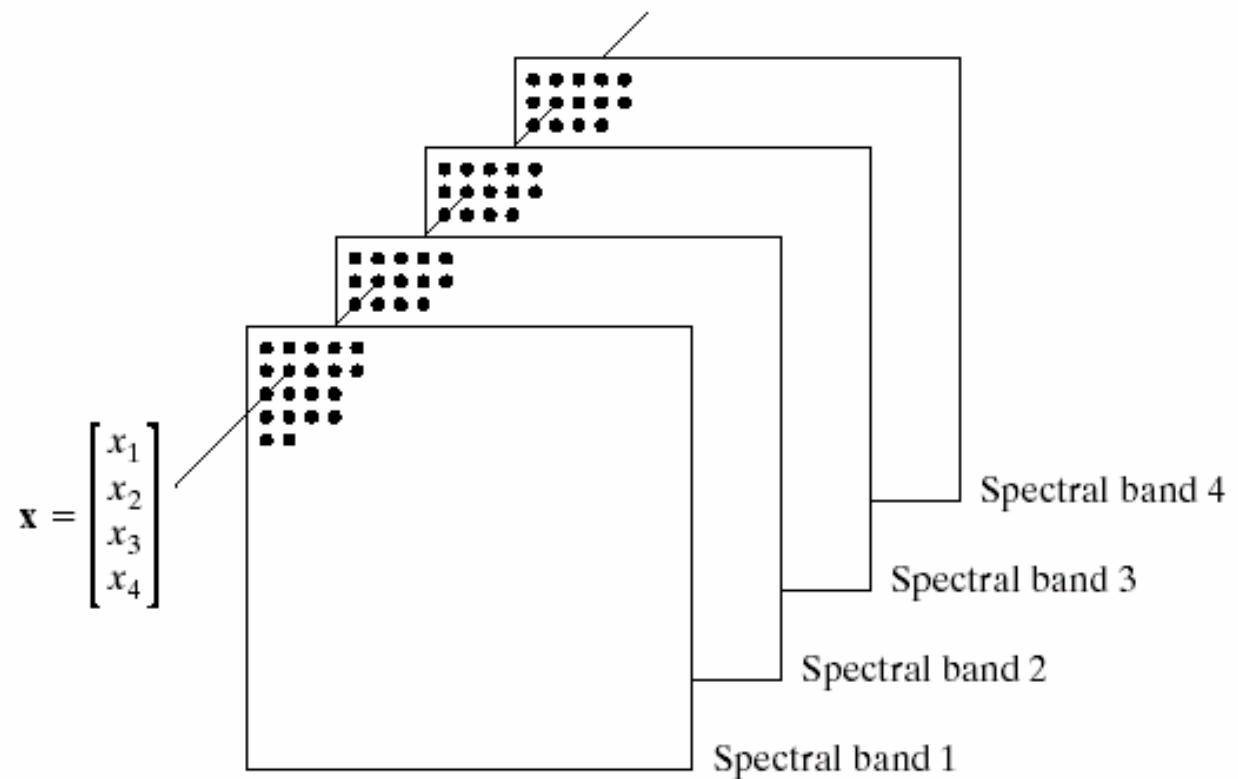


## Chapter 12

# Object Recognition

**FIGURE 12.12**

Formation of a pattern vector from registered pixels of four digital images generated by a multispectral scanner.

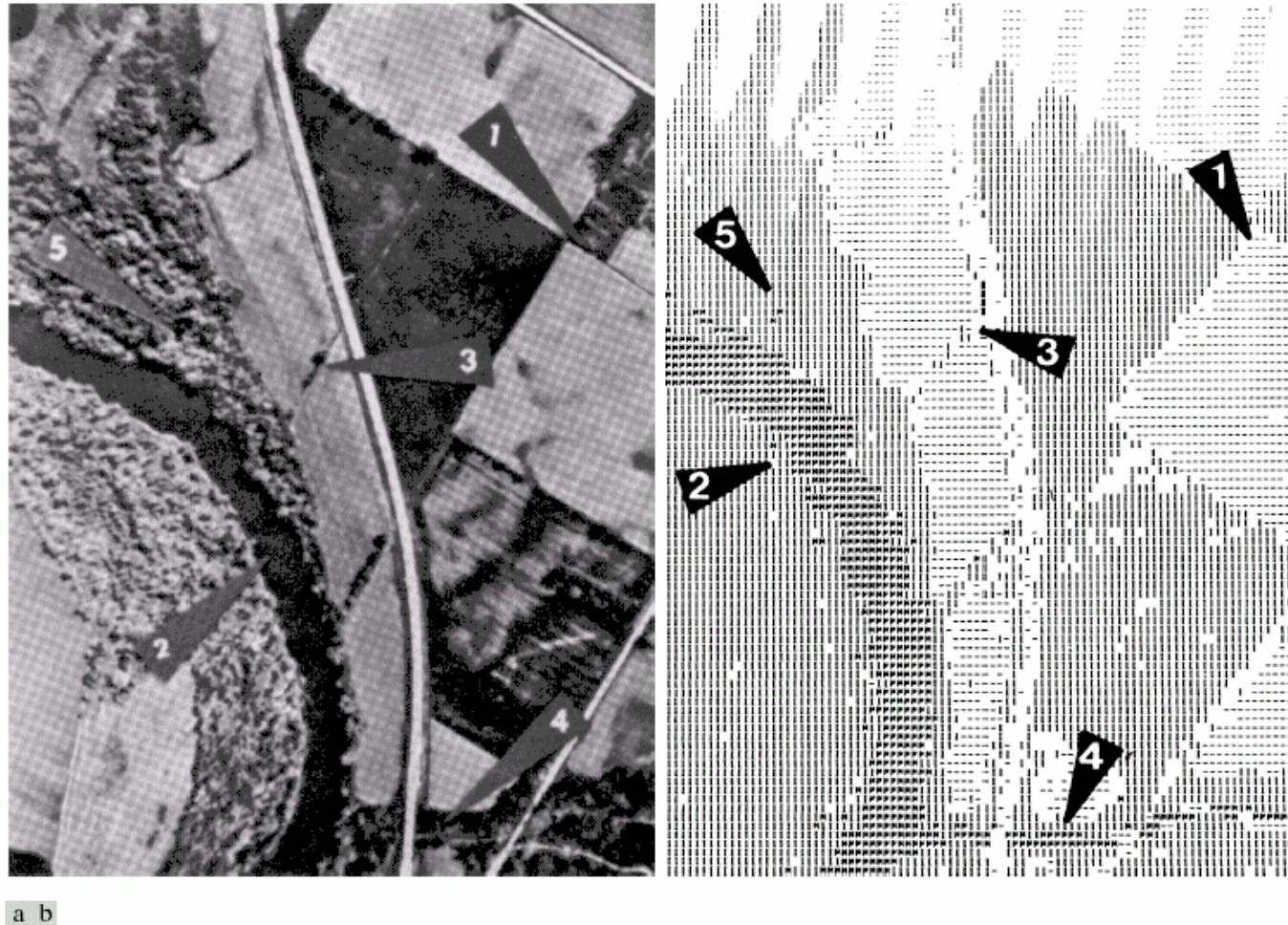






## Chapter 12

# Object Recognition



**FIGURE 12.13** (a) Multispectral image. (b) Printout of machine classification results using a Bayes classifier. (Courtesy of the Laboratory for Applications of Remote Sensing, Purdue University.)





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# Neural Network

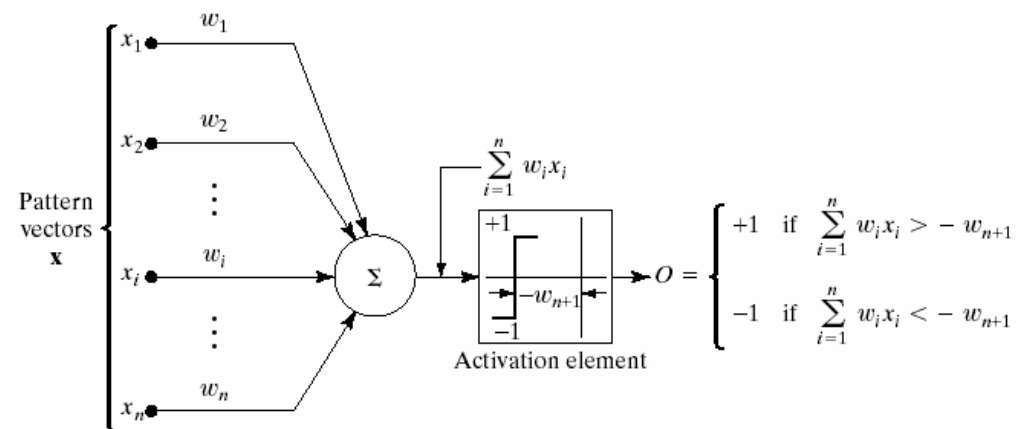
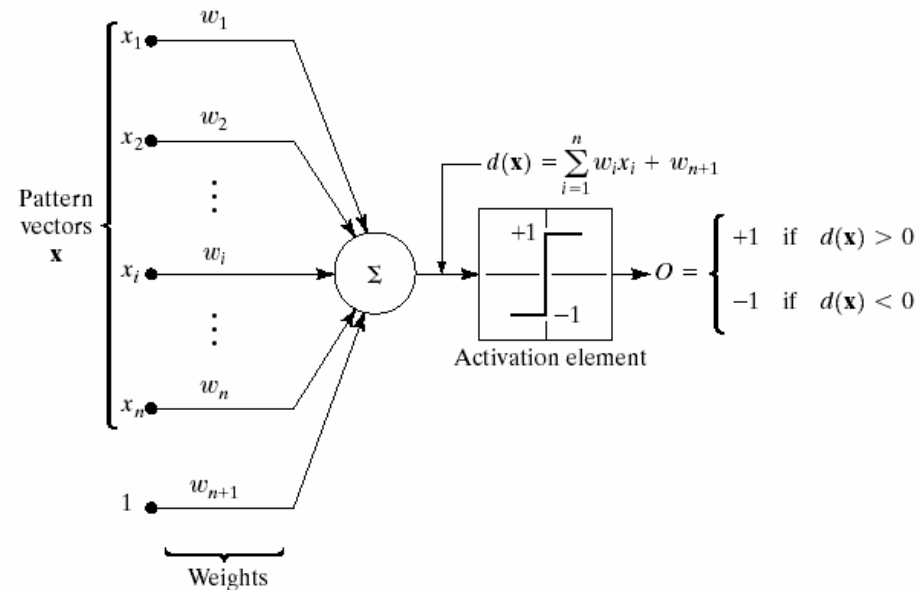
$$w(k+1) = w(k) - \alpha \left[ \frac{\partial J(w)}{\partial w} \right]_{w=w(k)}$$



## Chapter 12 Object Recognition

a  
b

**FIGURE 12.14**  
Two equivalent  
representations of  
the perceptron  
model for two  
pattern classes.





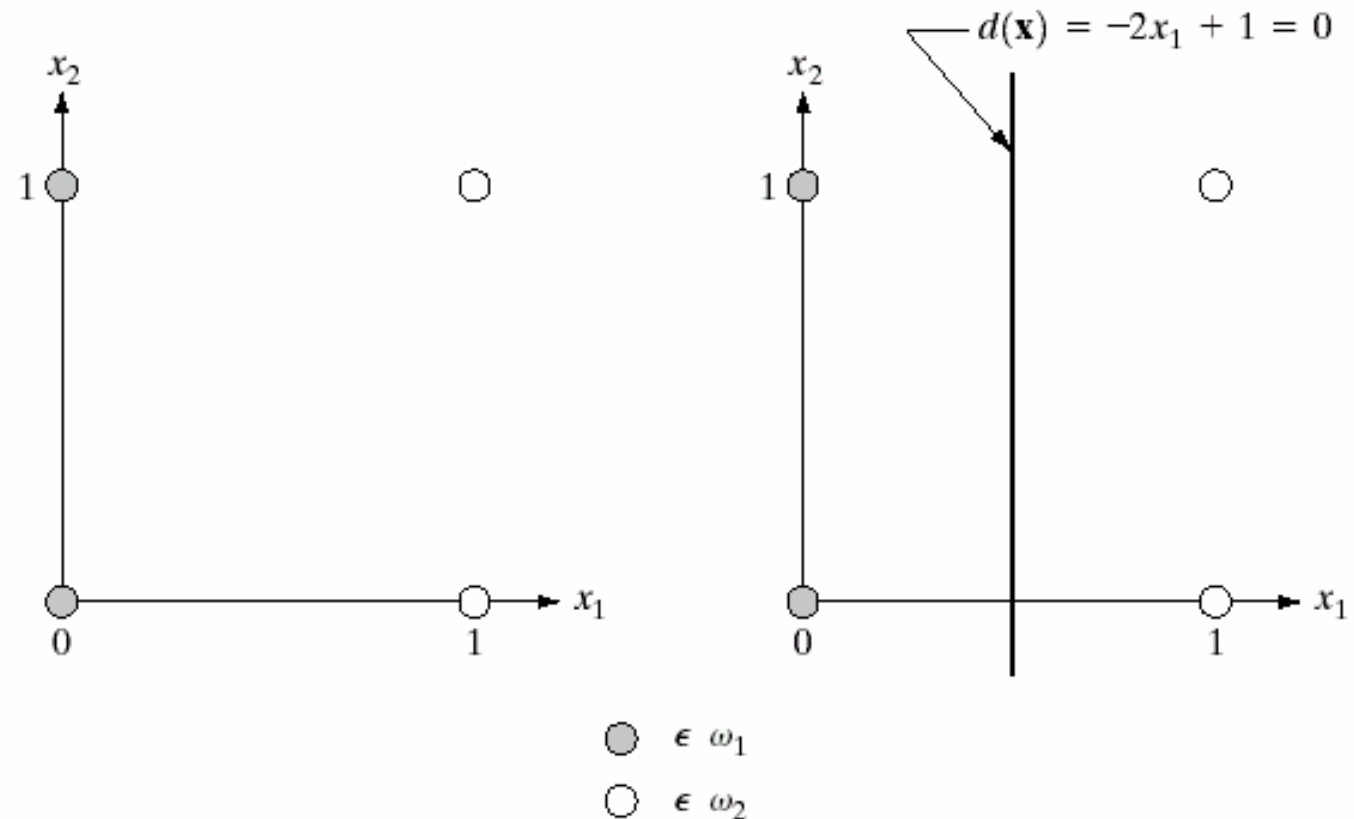
## Chapter 12

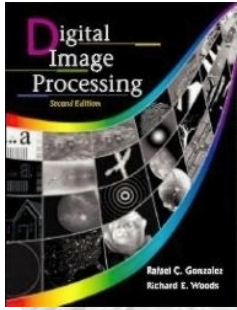
# Object Recognition

a b

**FIGURE 12.15**

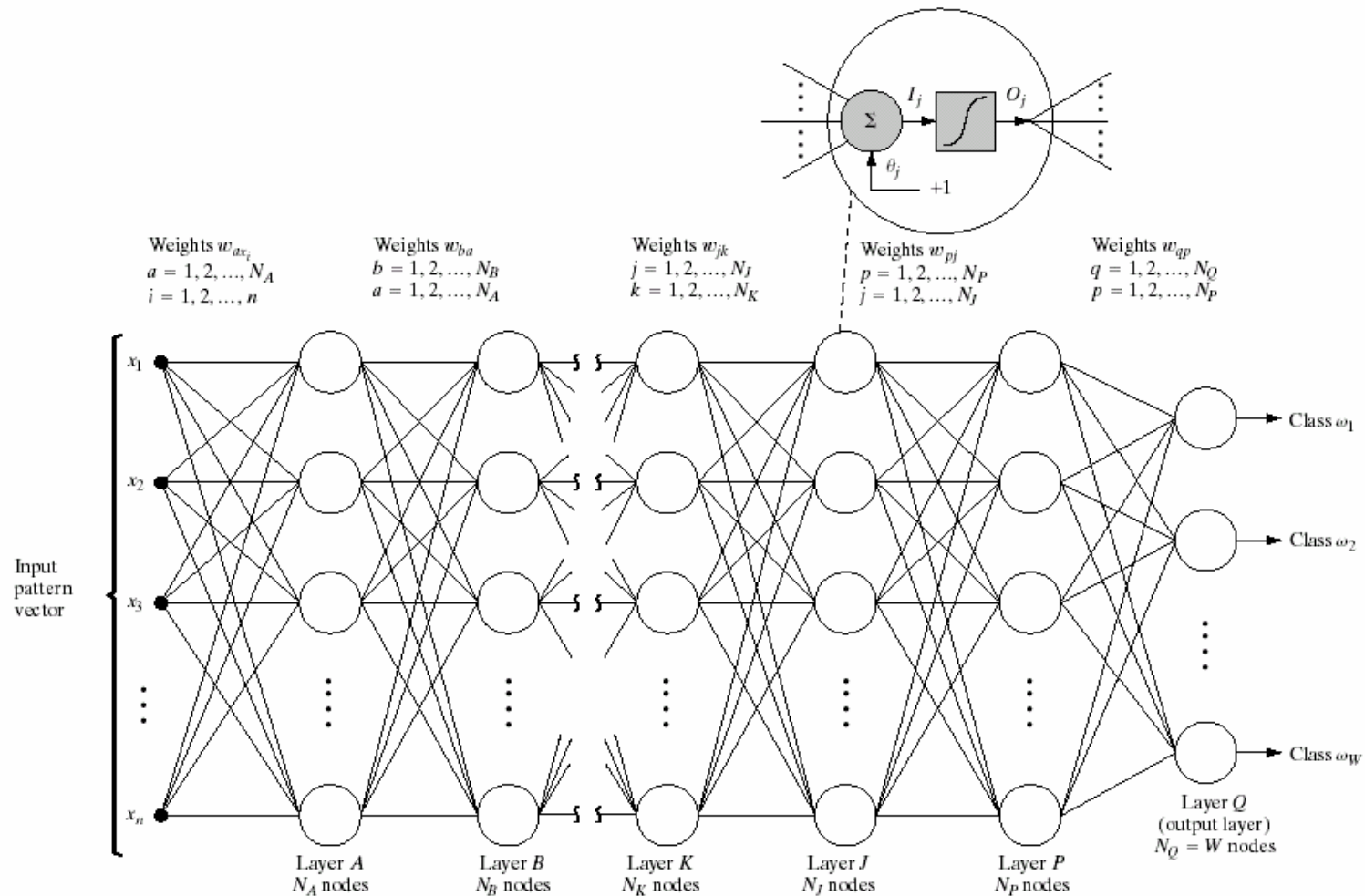
(a) Patterns belonging to two classes.  
(b) Decision boundary determined by training.



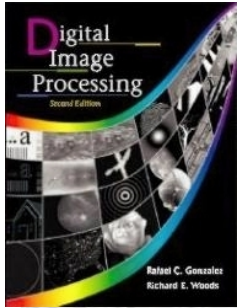


# Chapter 12

## Object Recognition

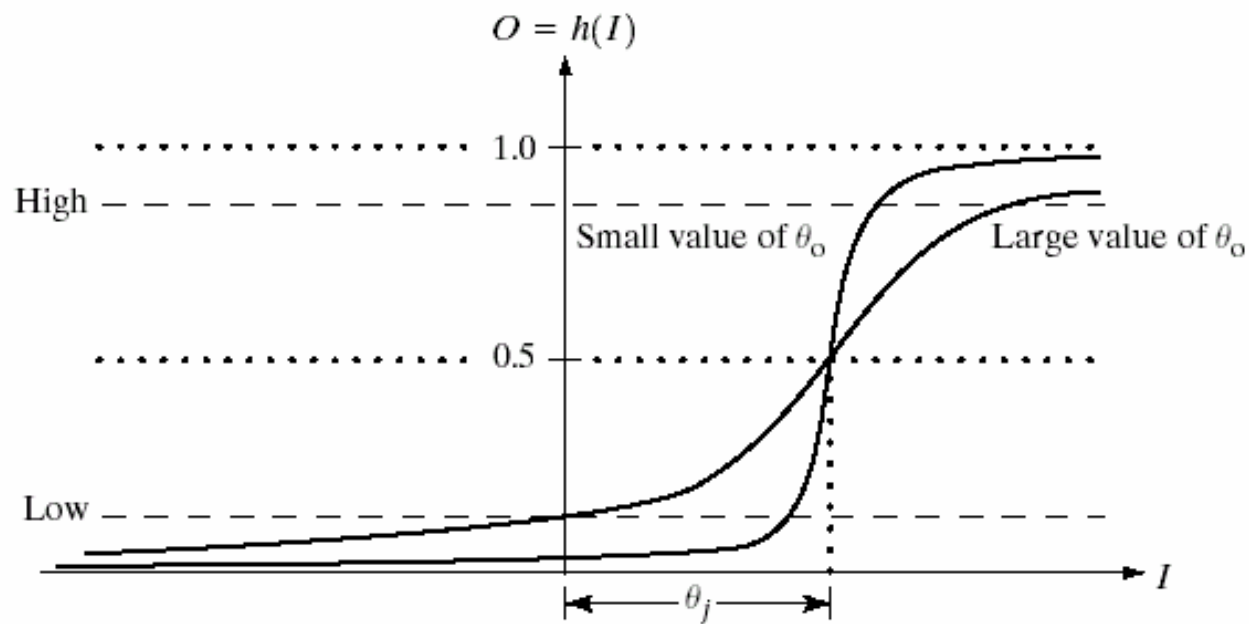


**FIGURE 12.16** Multilayer feedforward neural network model. The blowup shows the basic structure of each neuron element throughout the network. The offset,  $\theta_j$ , is treated as just another weight.

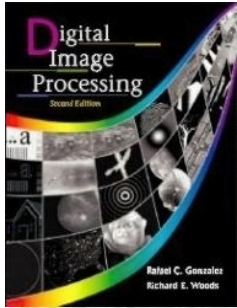


## Chapter 12

# Object Recognition



**FIGURE 12.17** The sigmoidal activation function of Eq. (12.2-47).



## Chapter 12

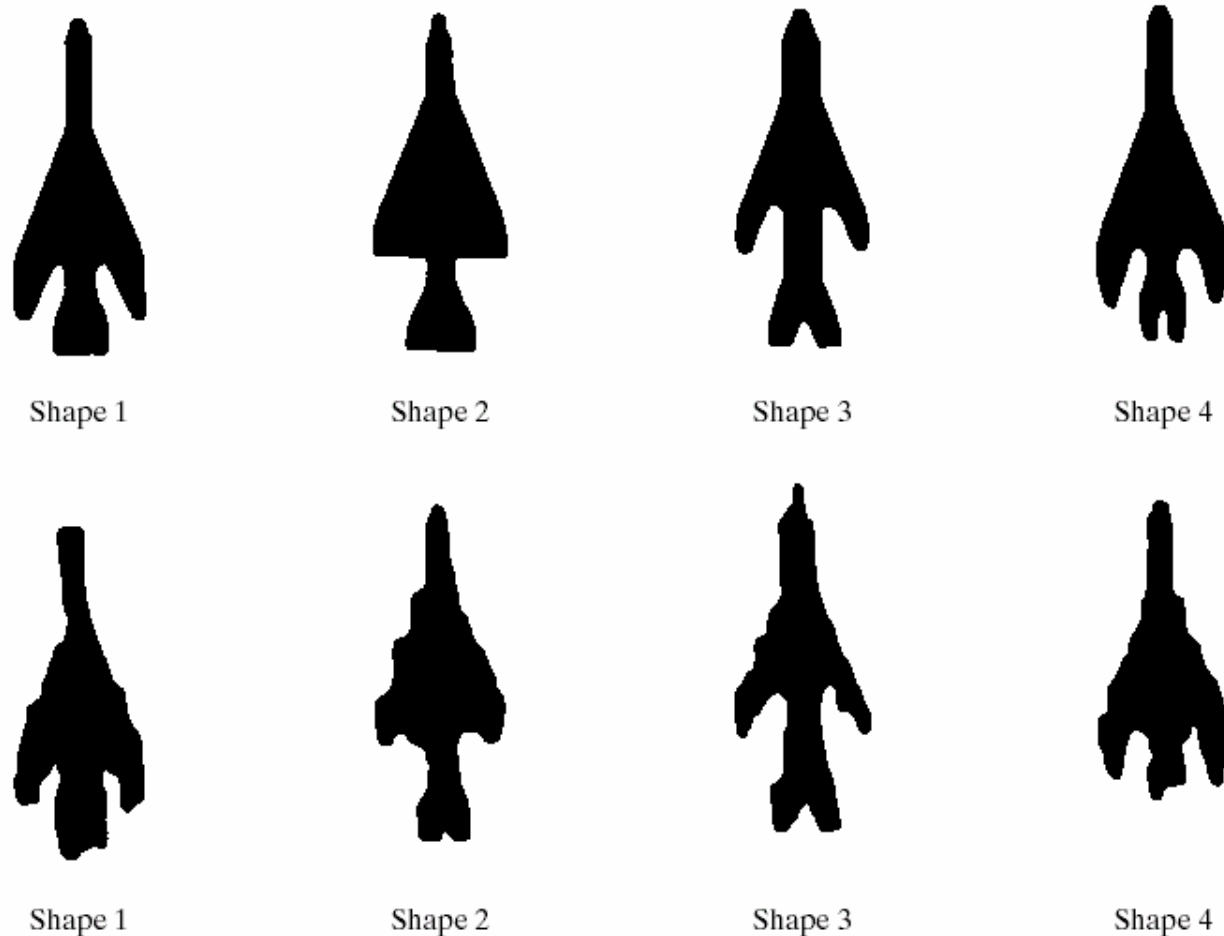
# Object Recognition

a  
b

**FIGURE 12.18**

(a) Reference shapes and  
(b) typical noisy shapes used in  
training the neural network of  
Fig. 12.19.

(Courtesy of Dr.  
Lalit Gupta, ECE  
Department,  
Southern Illinois  
University.)

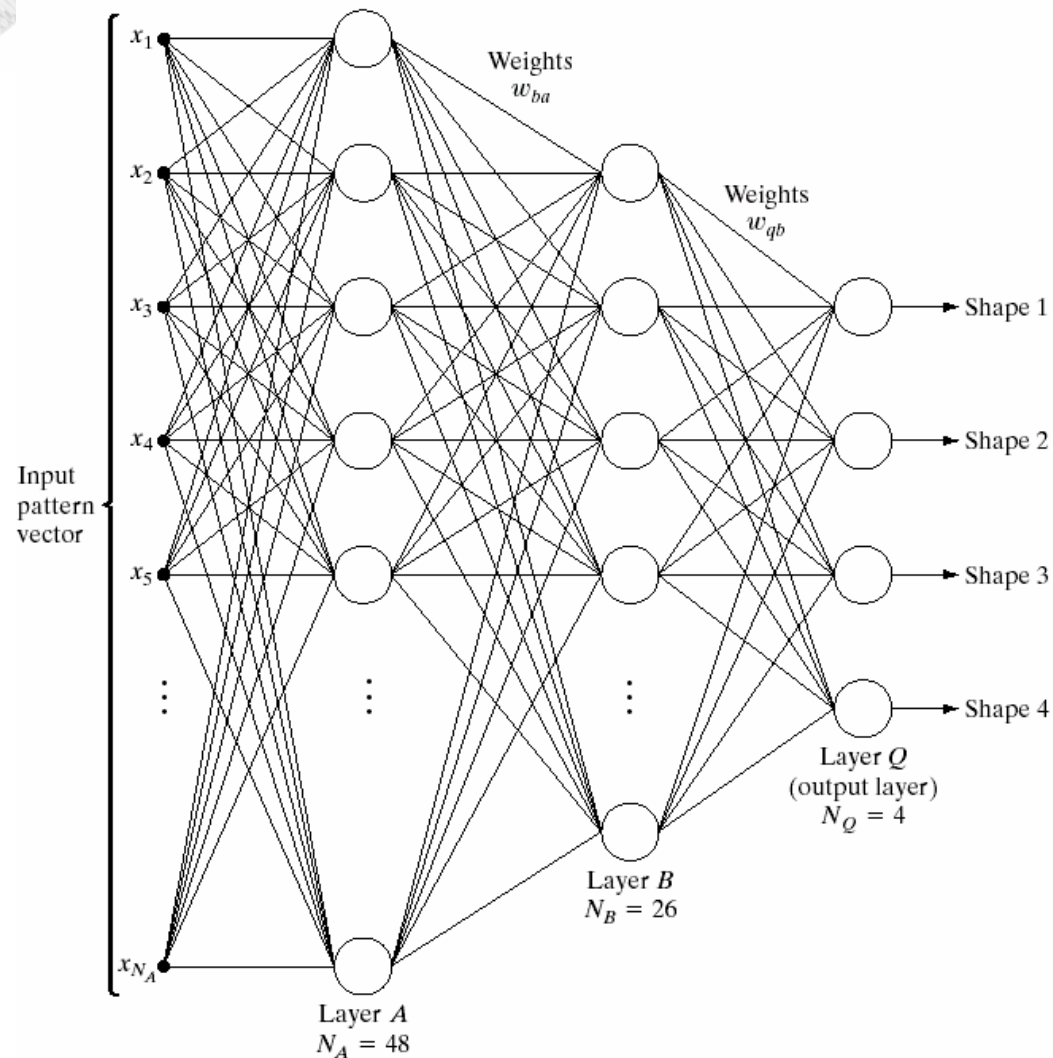






## Chapter 12

# Object Recognition



**FIGURE 12.19**

Three-layer neural network used to recognize the shapes in Fig. 12.18. (Courtesy of Dr. Lalit Gupta, ECE Department, Southern Illinois University.)

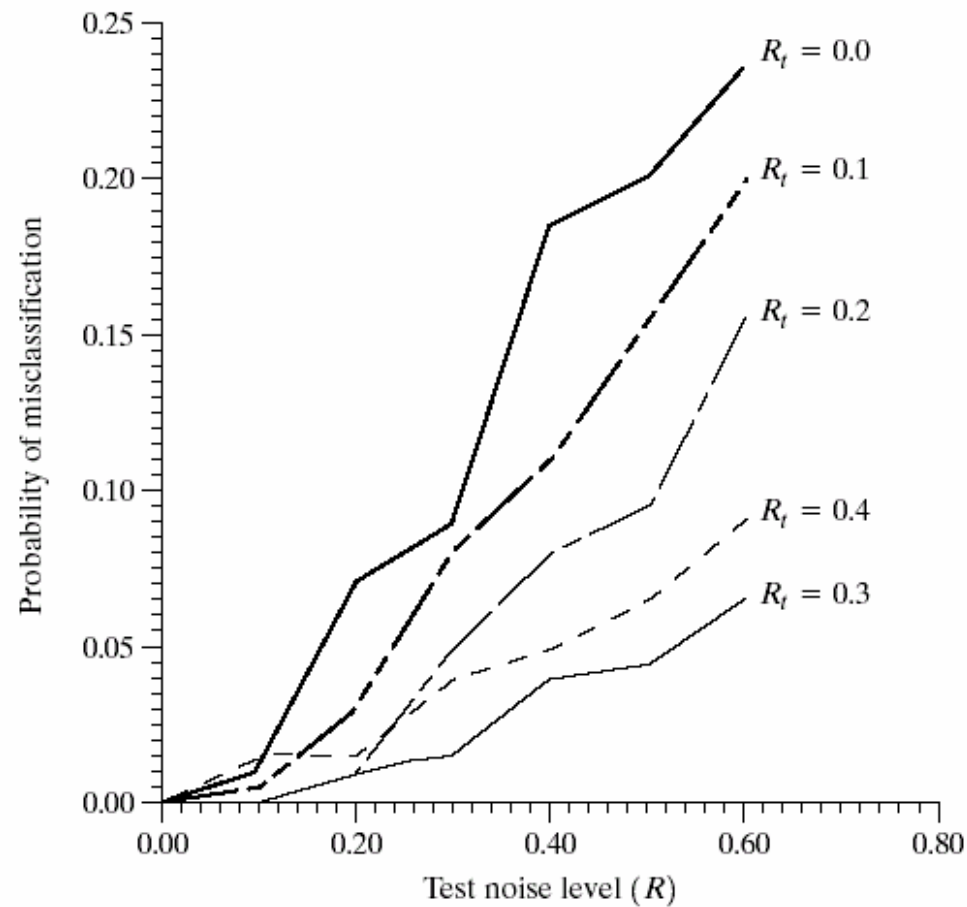


## Chapter 12

# Object Recognition

**FIGURE 12.20**

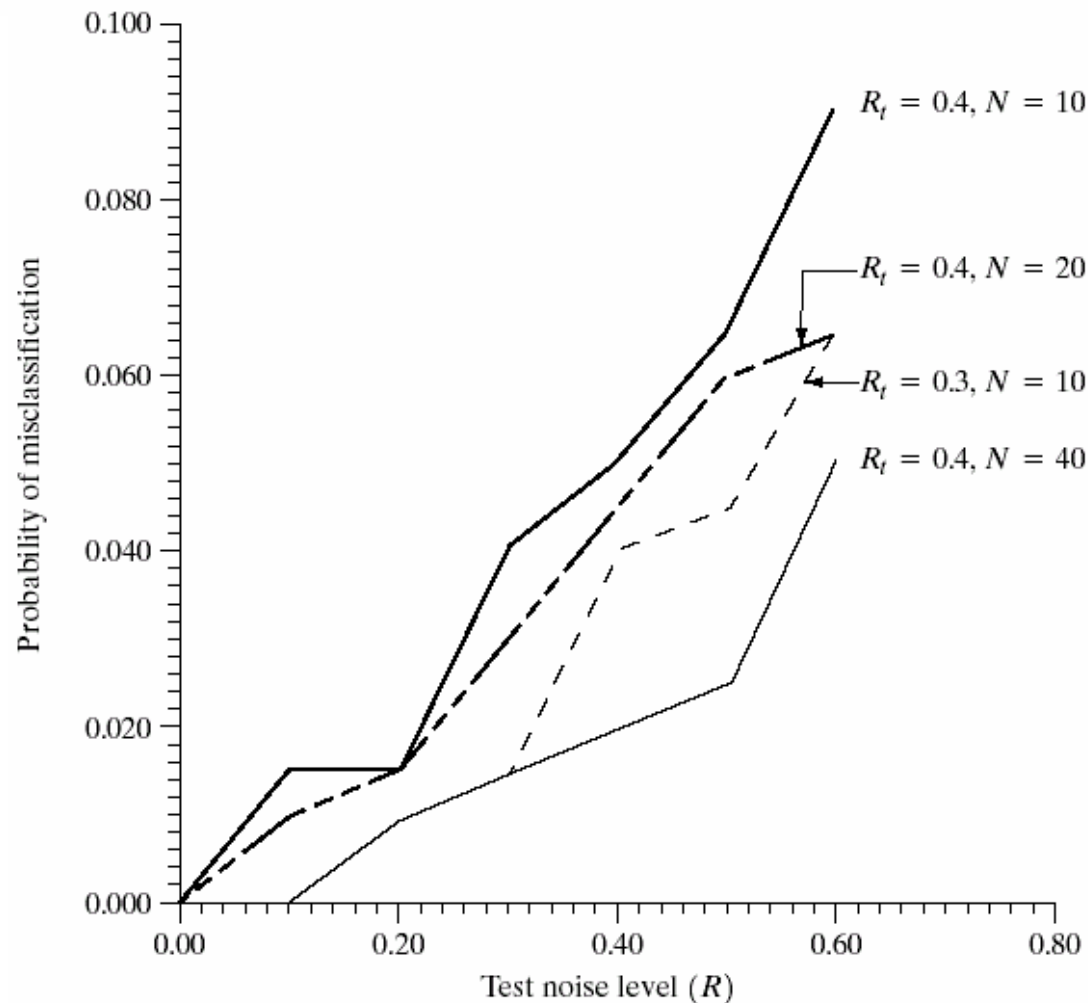
Performance of the neural network as a function of noise level. (Courtesy of Dr. Lalit Gupta, ECE Department, Southern Illinois University.)





## Chapter 12

### Object Recognition

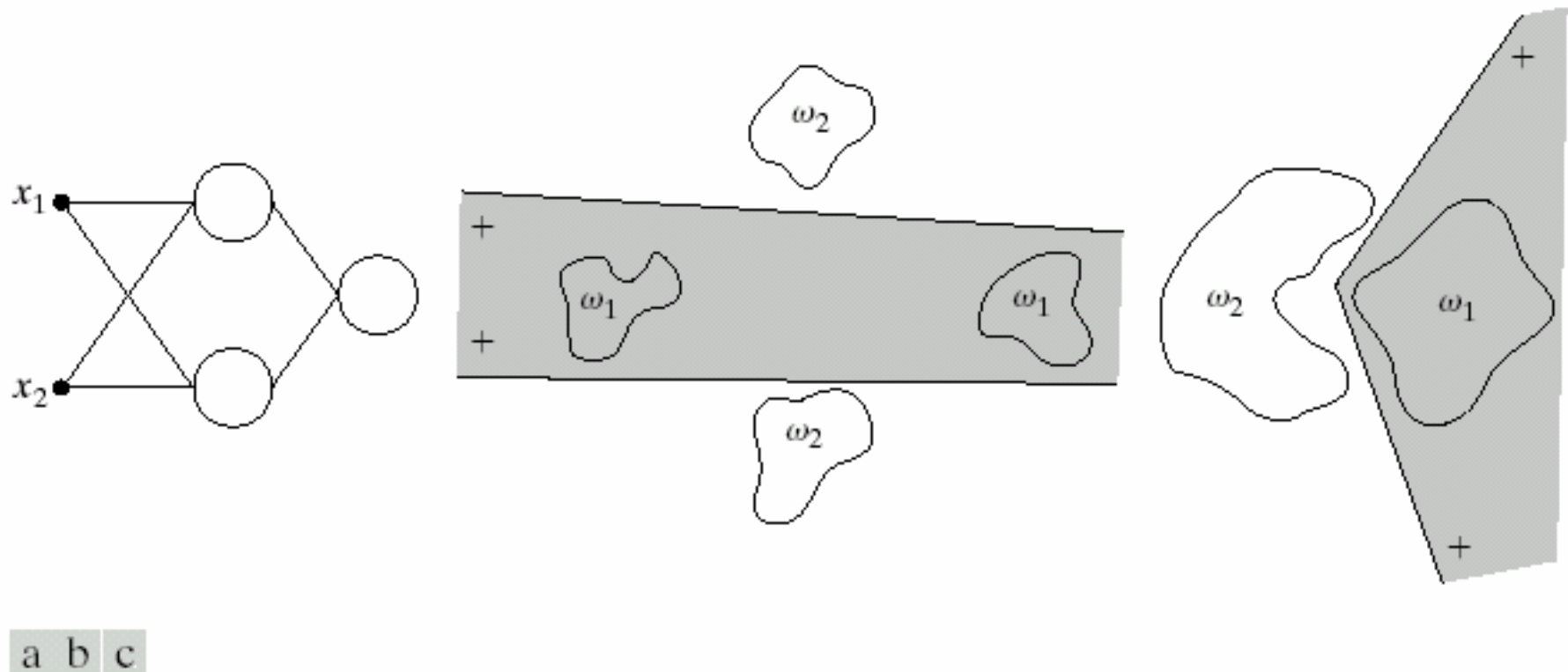


**FIGURE 12.21**

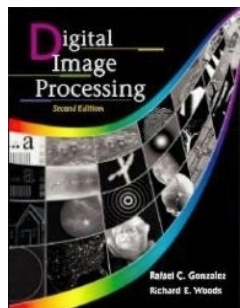
Improvement in performance for  $R_t = 0.4$  by increasing the number of training patterns (the curve for  $R_t = 0.3$  is shown for reference). (Courtesy of Dr. Lalit Gupta, ECE Department, Southern Illinois University.)



## Chapter 12 Object Recognition


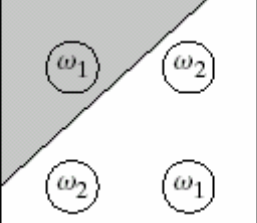
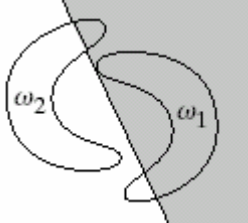

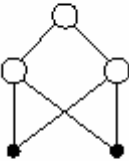
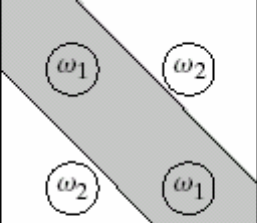
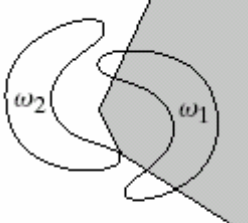
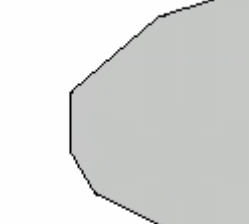
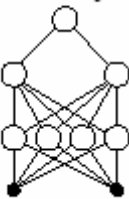
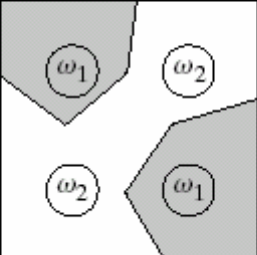
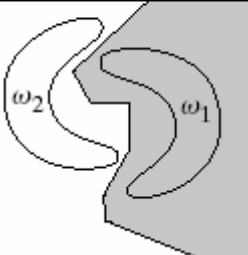
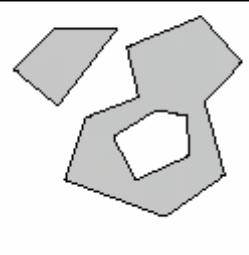


**FIGURE 12.22** (a) A two-input, two-layer, feedforward neural network. (b) and (c) Examples of decision boundaries that can be implemented with this network.

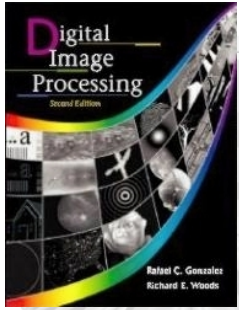


## Chapter 12

# Object Recognition

Network structure	Type of decision region	Solution to exclusive-OR problem	Classes with meshed regions	Most general decision surface shapes
Single layer 	Single hyperplane			
Two layers 	Open or closed convex regions			
Three layers 	Arbitrary (complexity limited by the number of nodes)			

**FIGURE 12.23**  
Types of decision regions that can be formed by single- and multilayer feed-forward networks with one and two layers of hidden units and two inputs. (Lippman)



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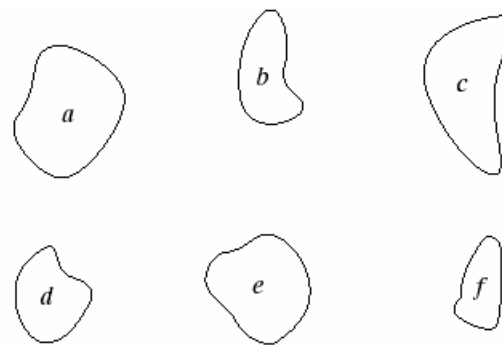
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# Structural Method



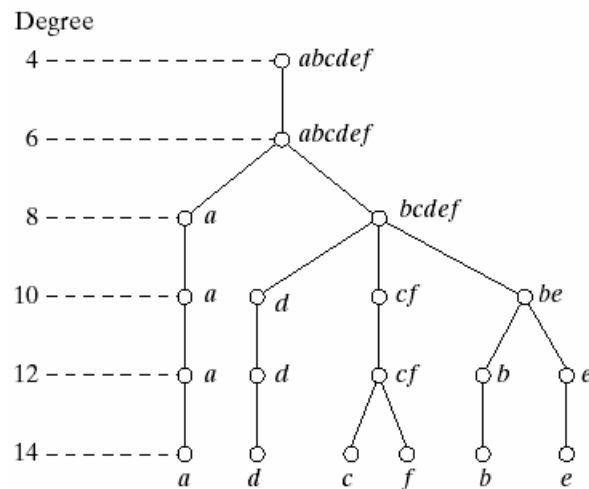


# Chapter 12 Object Recognition



**FIGURE 12.24**

(a) Shapes.  
(b) Hypothetical  
similarity tree.  
(c) Similarity  
matrix. (Bribiesca  
and Guzman.)



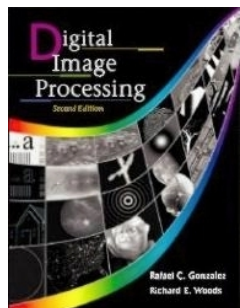
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>
<i>a</i>	$\infty$	6	6	6	6	6
<i>b</i>		$\infty$	8	8	10	8
<i>c</i>			$\infty$	8	8	12
<i>d</i>				$\infty$	8	8
<i>e</i>					$\infty$	8
<i>f</i>						$\infty$



# *Digital Image Processing, 2nd ed.*

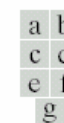
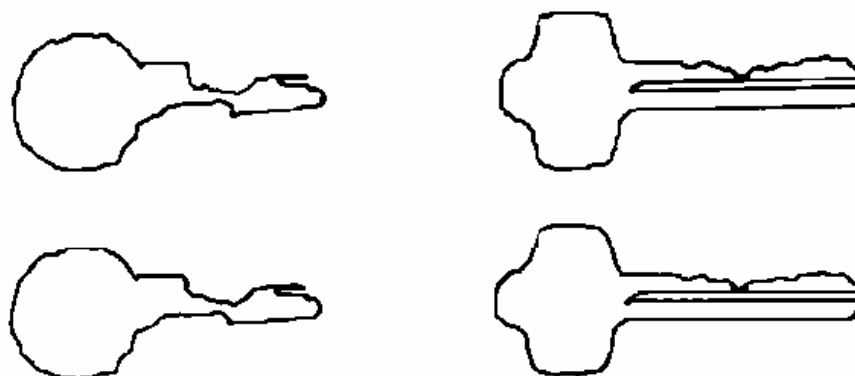
# String Matching

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## Chapter 12

# Object Recognition



**FIGURE 12.25** (a) and (b) Sample boundaries of two different object classes; (c) and (d) their corresponding polygonal approximations; (e)–(g) tabulations of  $R$ . (Sze and Yang.)

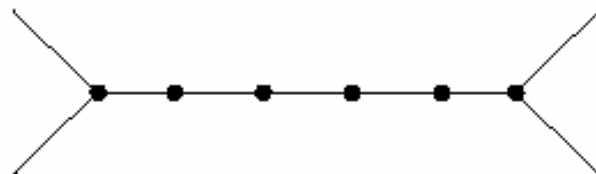
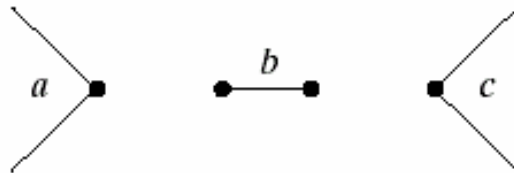
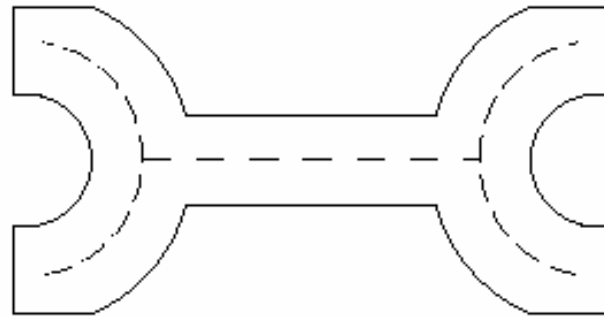
$R$	1.a	1.b	1.c	1.d	1.e	1.f
1.a	$\infty$					
1.b	16.0	$\infty$				
1.c	9.6	26.3	$\infty$			
1.d	5.1	8.1	10.3	$\infty$		
1.e	4.7	7.2	10.3	14.2	$\infty$	
1.f	4.7	7.2	10.3	8.4	23.7	$\infty$

$R$	2.a	2.b	2.c	2.d	2.e	2.f
2.a	$\infty$					
2.b	33.5	$\infty$				
2.c	4.8	5.8	$\infty$			
2.d	3.6	4.2	19.3	$\infty$		
2.e	2.8	3.3	9.2	18.3	$\infty$	
2.f	2.6	3.0	7.7	13.5	27.0	$\infty$

$R$	1.a	1.b	1.c	1.d	1.e	1.f
2.a	1.24	1.50	1.32	1.47	1.55	1.48
2.b	1.18	1.43	1.32	1.47	1.55	1.48
2.c	1.02	1.18	1.19	1.32	1.39	1.48
2.d	1.02	1.18	1.19	1.32	1.29	1.40
2.e	0.93	1.07	1.08	1.19	1.24	1.25
2.f	0.89	1.02	1.02	1.24	1.22	1.18



## Chapter 12 Object Recognition



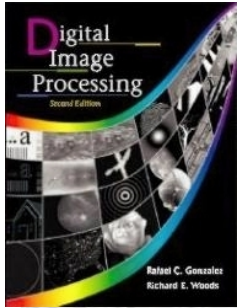
a  
b  
c

**FIGURE 12.26**

(a) Object represented by its (pruned) skeleton.

(b) Primitives.

(c) Structure generated by using a regular string grammar.

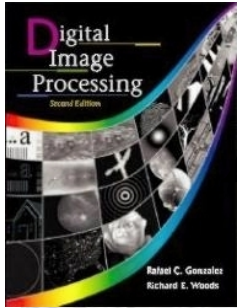


## Chapter 12

# Object Recognition

**TABLE 12.1**  
Example of  
semantic  
information  
attached to  
production rules.

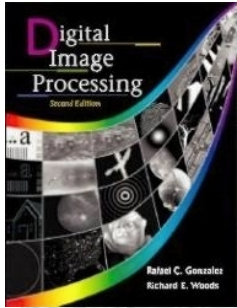
Production	Semantic Information
$S \rightarrow aA$	Connections to $a$ are made only at the dot. The direction of $a$ , denoted $\theta$ , is given by the direction of the perpendicular bisector of the line joining the end points of the two undotted segments. The line segments are 3 cm each.
$A \rightarrow bA$	Connections to $b$ are made only at the dots. No multiple connections are allowed. The direction of $b$ must be the same as the direction of $a$ . The length of $b$ is 0.25 cm. This production cannot be applied more than 10 times.
$A \rightarrow bB$	The direction of $a$ and $b$ must be the same. Connections must be simple and made only at the dots.
$B \rightarrow c$	The direction of $c$ and $a$ must be the same. Connections must be simple and made only at the dots.



*Digital Image Processing, 2nd ed.*

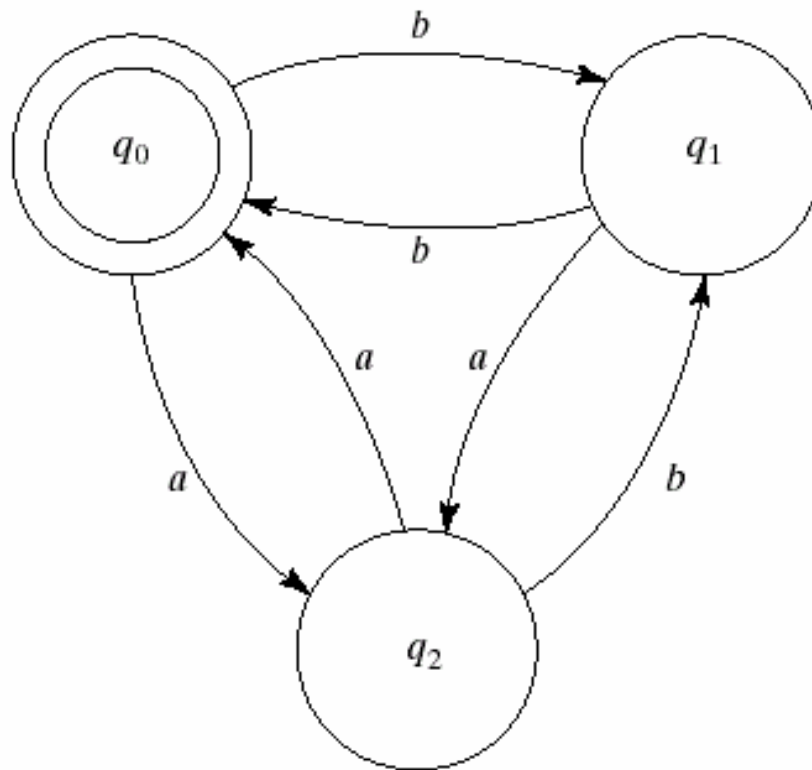
[www.imageprocessingbook.com](http://www.imageprocessingbook.com)

# Automatic as string recognizers



## Chapter 12

# Object Recognition



**FIGURE 12.27** A finite automaton.





*Digital Image Processing, 2nd ed.*

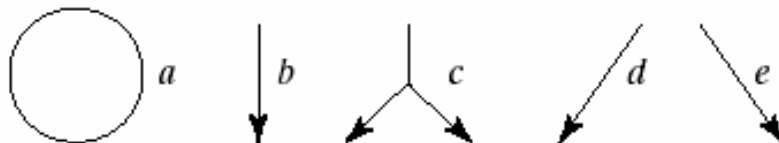
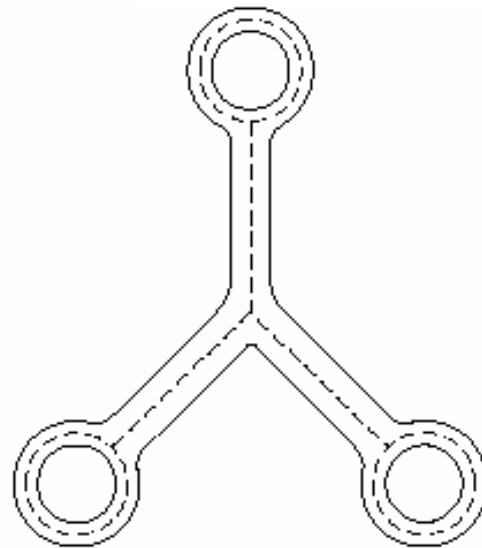
[www.imageprocessingbook.com](http://www.imageprocessingbook.com)

# Syntactic Recognition of Trees



## Chapter 12

# Object Recognition



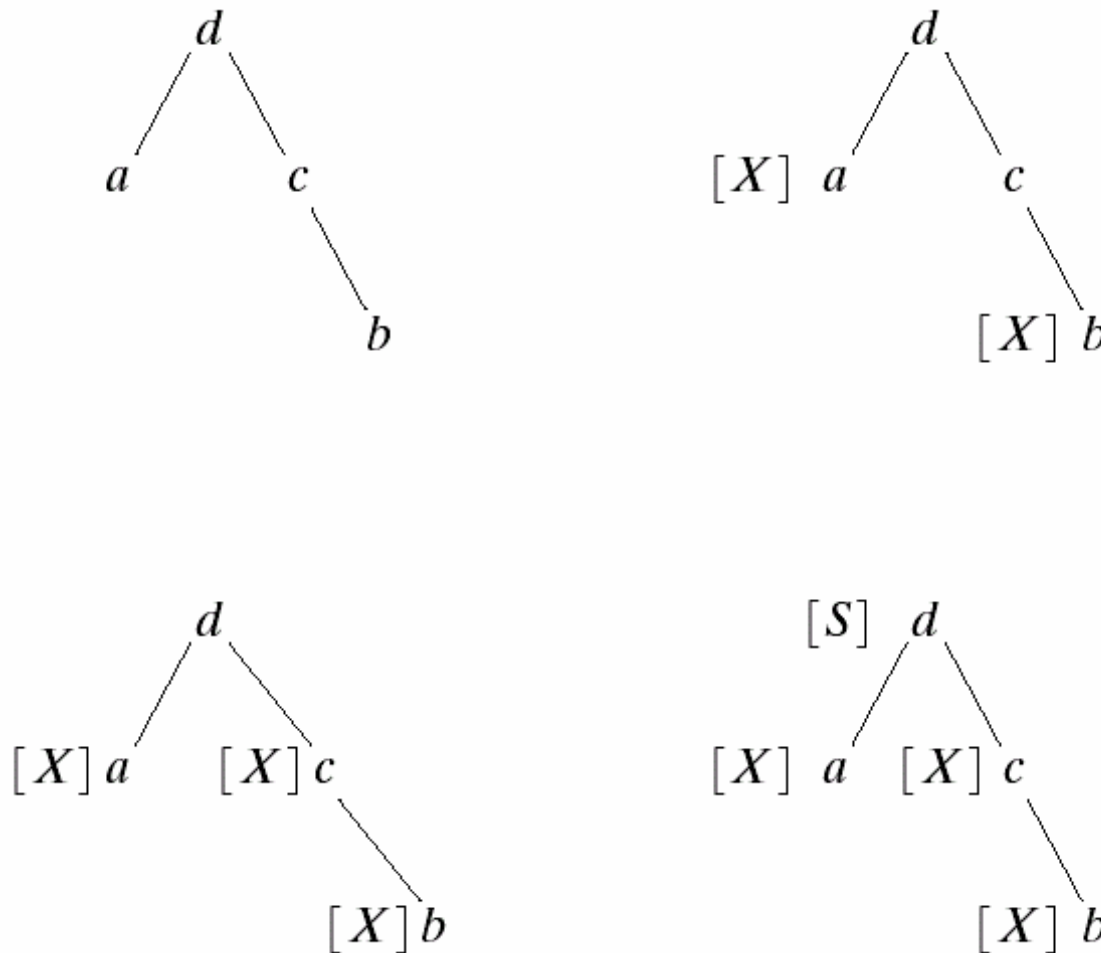
a  
b

**FIGURE 12.28**

(a) An object and  
(b) primitives  
used for  
representing the  
skeleton by  
means of a tree  
grammar.

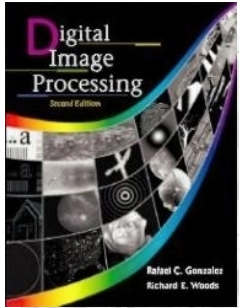


## Chapter 12 Object Recognition



a	b
c	d

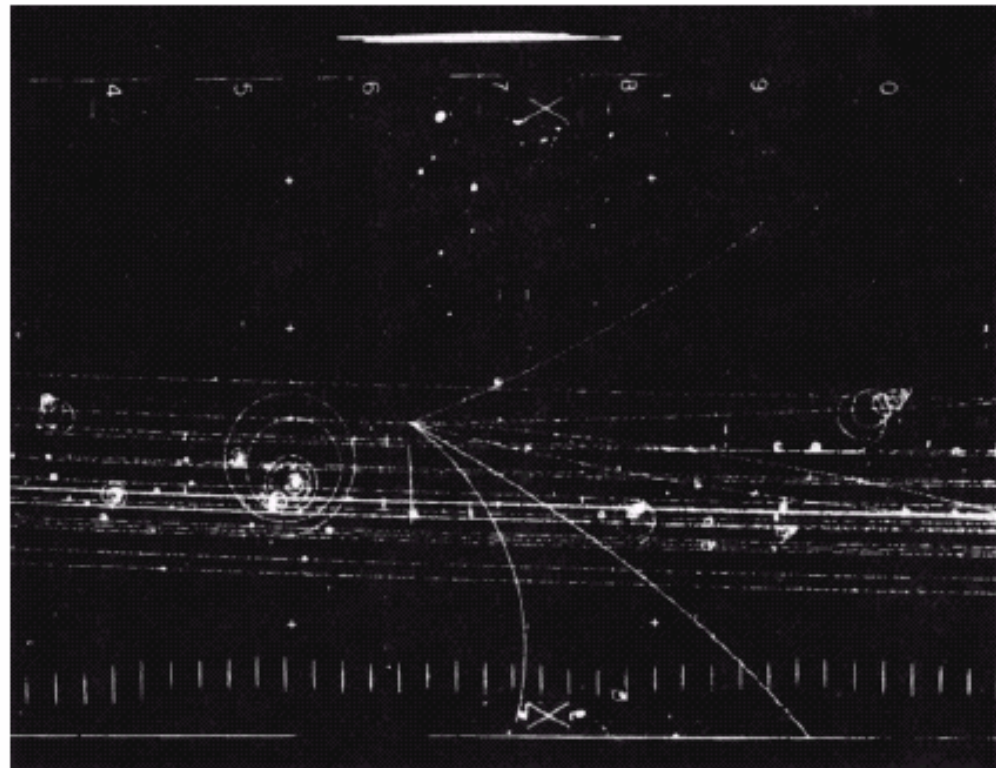
**FIGURE 12.29**  
Processing stages  
of a frontier-to-  
root tree  
automaton:  
(a) Input tree.  
(b) State  
assignment to  
frontier nodes.  
(c) State  
assignment to  
intermediate  
nodes. (d) State  
assignment to  
root node.

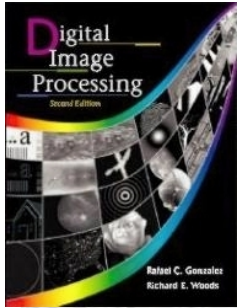


## Chapter 12

# Object Recognition

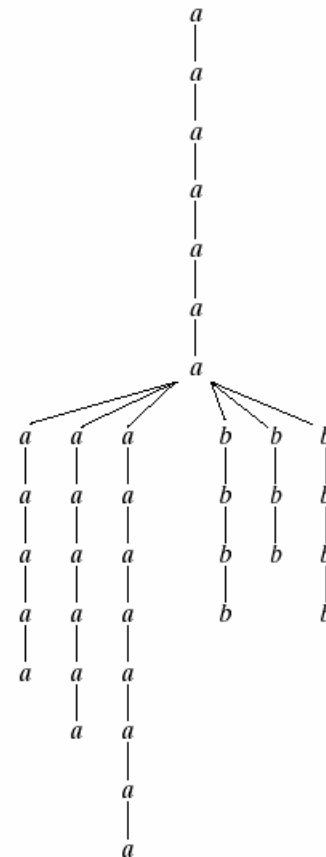
**FIGURE 12.30** A bubble chamber photograph. (Fu and Bhargava.)





# Chapter 12

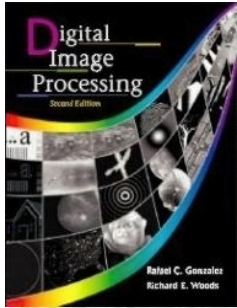
## Object Recognition



a b

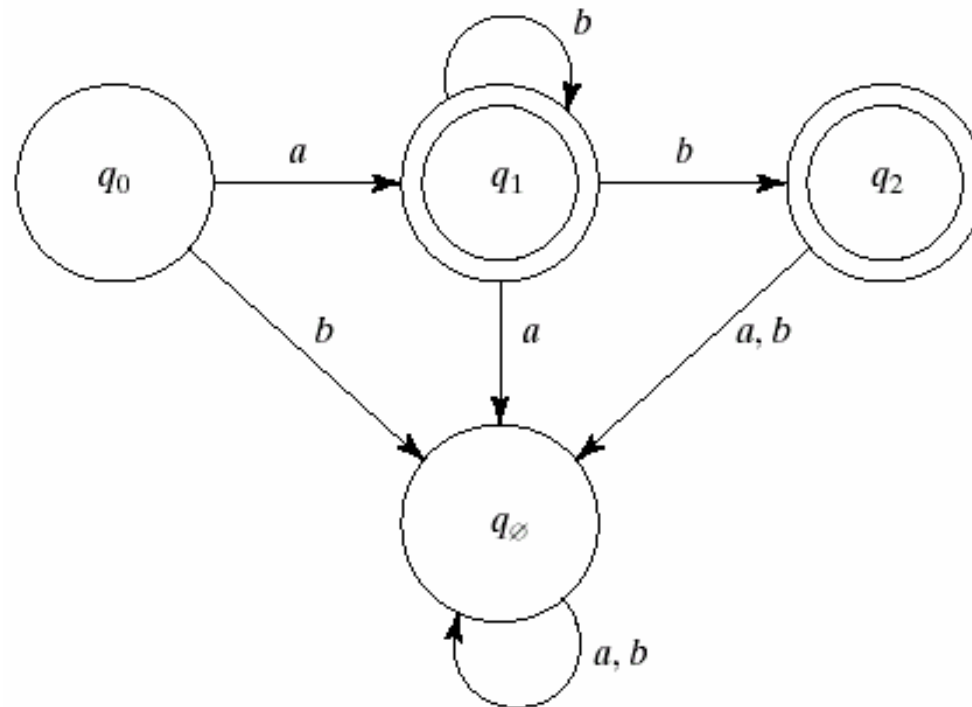
**FIGURE 12.31**

(a) Coded event from Fig. 12.30.  
(b) Corresponding tree representation.  
(Fu and Bhargava.)



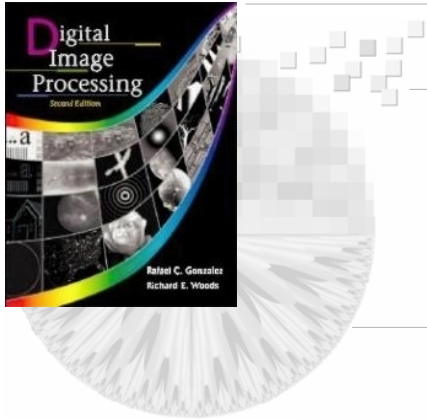
## Chapter 12

### Object Recognition



**FIGURE 12.32**

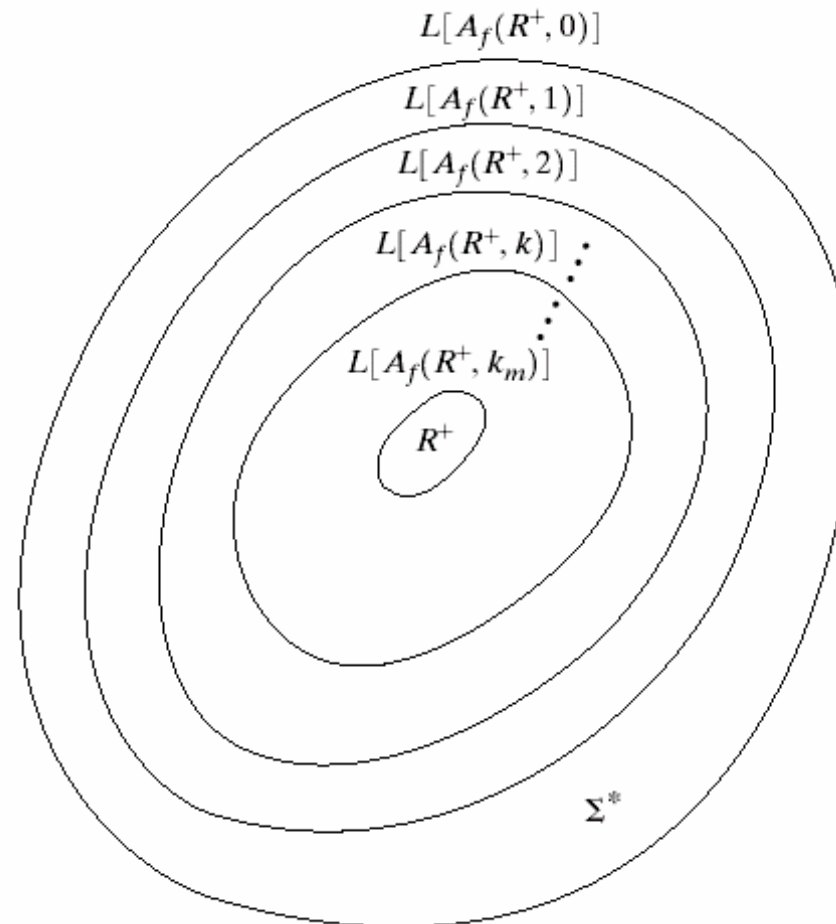
State diagram for the finite automaton inferred from the sample set  $R^+ = \{a, ab, abb\}$ .



# Chapter 12

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**FIGURE 12.33**  
Relationship  
between  
 $L[A_f(R^+, k)]$  and  
 $k$ . The value of  $k_m$   
is such that  
 $k_m \geq$  (length of  
the longest string  
in  $R^+$ ).

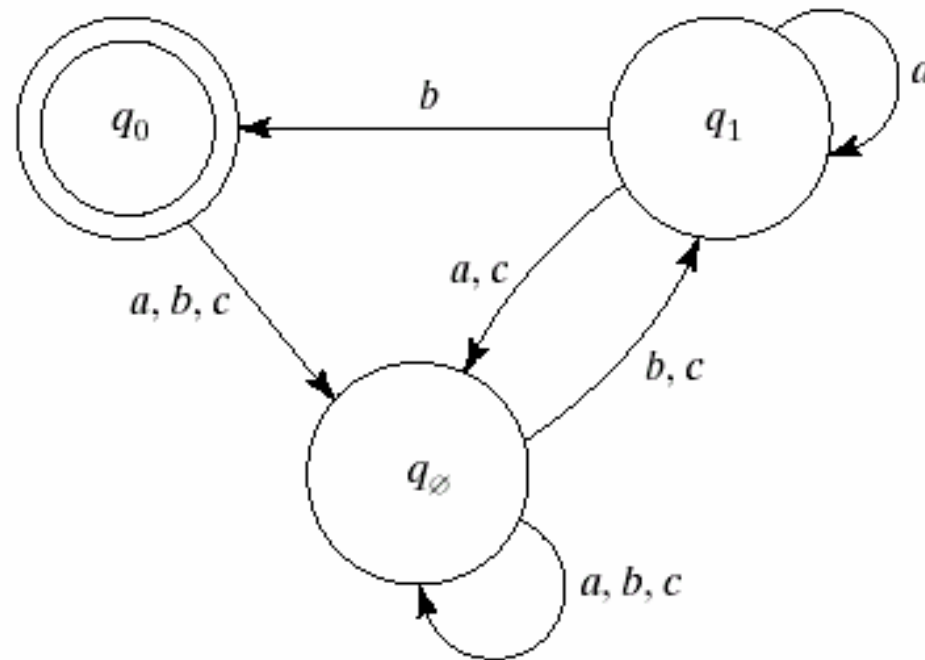






## Chapter 12

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**FIGURE 12.34** State diagram for the automaton  $A_f(R^+, 1)$  inferred from the sample set  $R^+ = \{caaab, bbaab, caab, bbab, cab, bbb, cb\}$ .