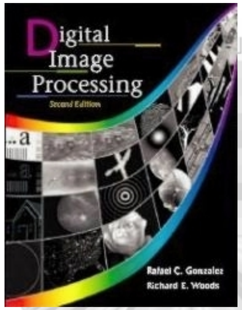


Chapter 5
Image Restoration

- 5.1 A mode of the Image Degradation/ Restoration Process
- Given g(x,y), some knowledge the degradation about the degradation function H, and some knowledge about the additive noise term $\eta(x,y)$, the objective of restoration is to obtain an estimate $\hat{f}(x,y)$ of original image.

- We want the estimate to be as close as possible to the original input image and, in general, the more we know about H and η ,the closer $\hat{f}(x, y)$ will be to $f(x, y)$
- The degraded image is given in the spatial domain by

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y) \quad (5.1\text{–}1)$$

- Where $h(x, y)$ the spatial representation of the degradation function ,the symbol "*" indicates spatial convolution.
- we may write the model in Eq(5.1-1) in an equivalent frequency domain representation:

$$G(u,v) = H(u,v)F(u,v) + N(u,v) \quad (5.1-2)$$

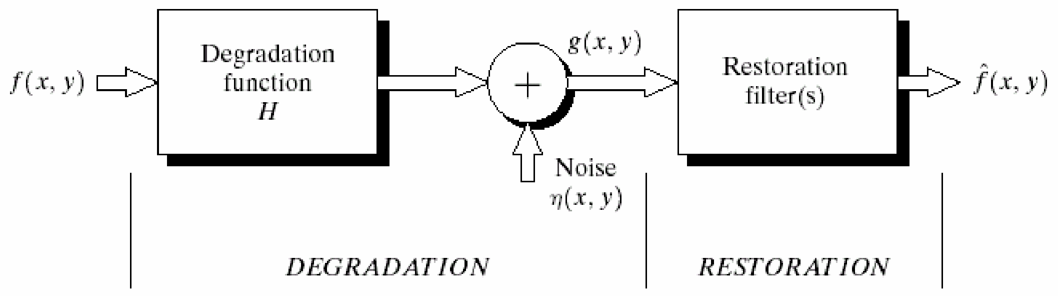


FIGURE 5.1 A model of the image degradation/restoration process.

5.2 Noise Models

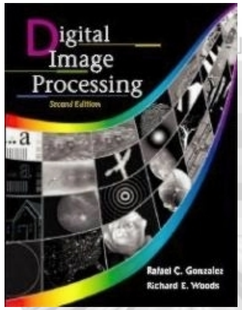
- 5.2.1 Spatial and Frequency Properties of Noise
- When the Fourier spectrum of noise is constant, the noise usually is called *white noise*.
- 5. 2. 2 Some Important Noise Probability Density Function
- **Gaussian noise**
- Gaussian (also called normal), noise models are used frequently in practice.

- The PDF of a Gaussian random variable,z, is given by

$$P(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\mu)^2/2\sigma^2} \quad (5.2-1)$$

- Where z represents gray level, μ is the mean of average value of z,and σ is its standard deviation.

- Approximately 70% of its values will be in the range $\left[(\mu-\sigma),(\mu+\sigma)\right]$,and about 95 % will be in the range $\left[(\mu-2\sigma),(\mu+2\sigma)\right]$



- Rayleigh noise
- The PDF of Rayleigh noise is given by

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases} \qquad (5.2-2)$$

- The mean and variance of this density are given by

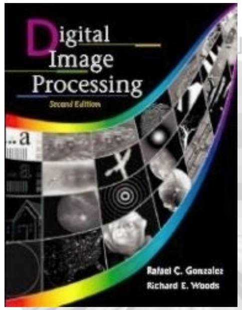
$$\mu = a + \sqrt{\pi b / 4} \qquad (5.2-3)$$

$$\sigma^2 = \frac{b(4-\pi)}{4} \qquad (5.2-4)$$

- The Rayleigh density can be quite useful for approximating skewed histograms.

- **Erlang (Gamma) noise**
- The PDF of Erlang noise is given by

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & for\ z \geq 0 \\ 0 & for\ z < 0 \end{cases} \qquad (5.2-5)$$



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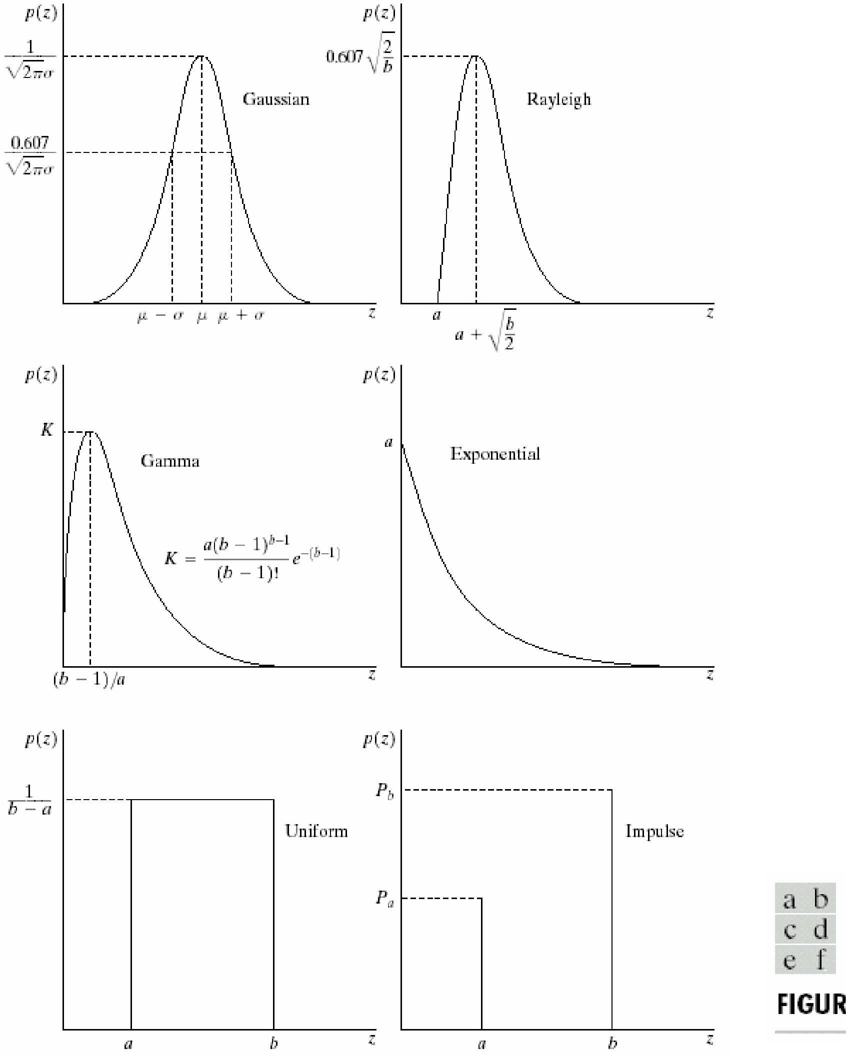

FIGURE 5.2 Some important probability density functions.

- Exponential noise
- The PDF of exponential noise is given by

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \qquad (5.2-8)$$

- Uniform noise
- The PDF of uniform noise is given by

$$p(z) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases} \qquad (5.2-11)$$

- Impulse (salt-and-pepper) noise
- The PDF of (bipolar) impulse noise is given by

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases} \qquad (5.2-14)$$

- Impulse noise values will resemble salt-and-pepper granules randomly distributed over the image.
- Impulse noise generally is digitized as extreme (pure black or white) values in an image.

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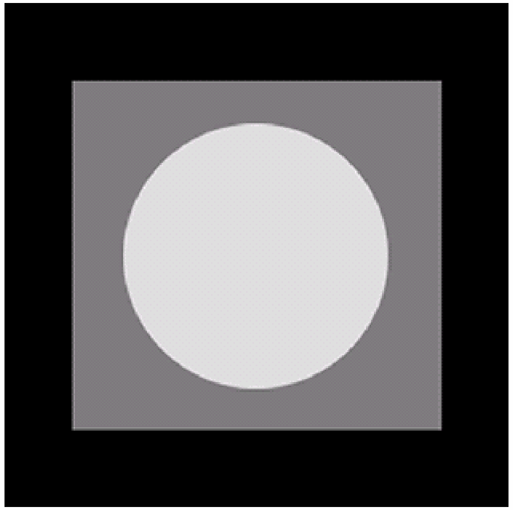

FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

- 5.2.3 Periodic Noise
- Periodic Noise in an image irises typically from electrical or electromechanical interference during image acquisition. This is the only type of spatially dependent noise that will be considered in this chapter.
- Periodic noise can be reduced significantly via frequency domain filtering.
- The Fourier transform of a pure sinusoid is a pair of conjugate impulses located at the conjugate frequencies of the sine wave .

- 5.2.4 Estimation of Noise Parameters
- The parameters of periodic noise typically are estimated by inspection of the Fourier spectrum of the image.
- As noted in the previous section, periodic noise tends to produce frequency spikes that often can be detected even by visual analysis.

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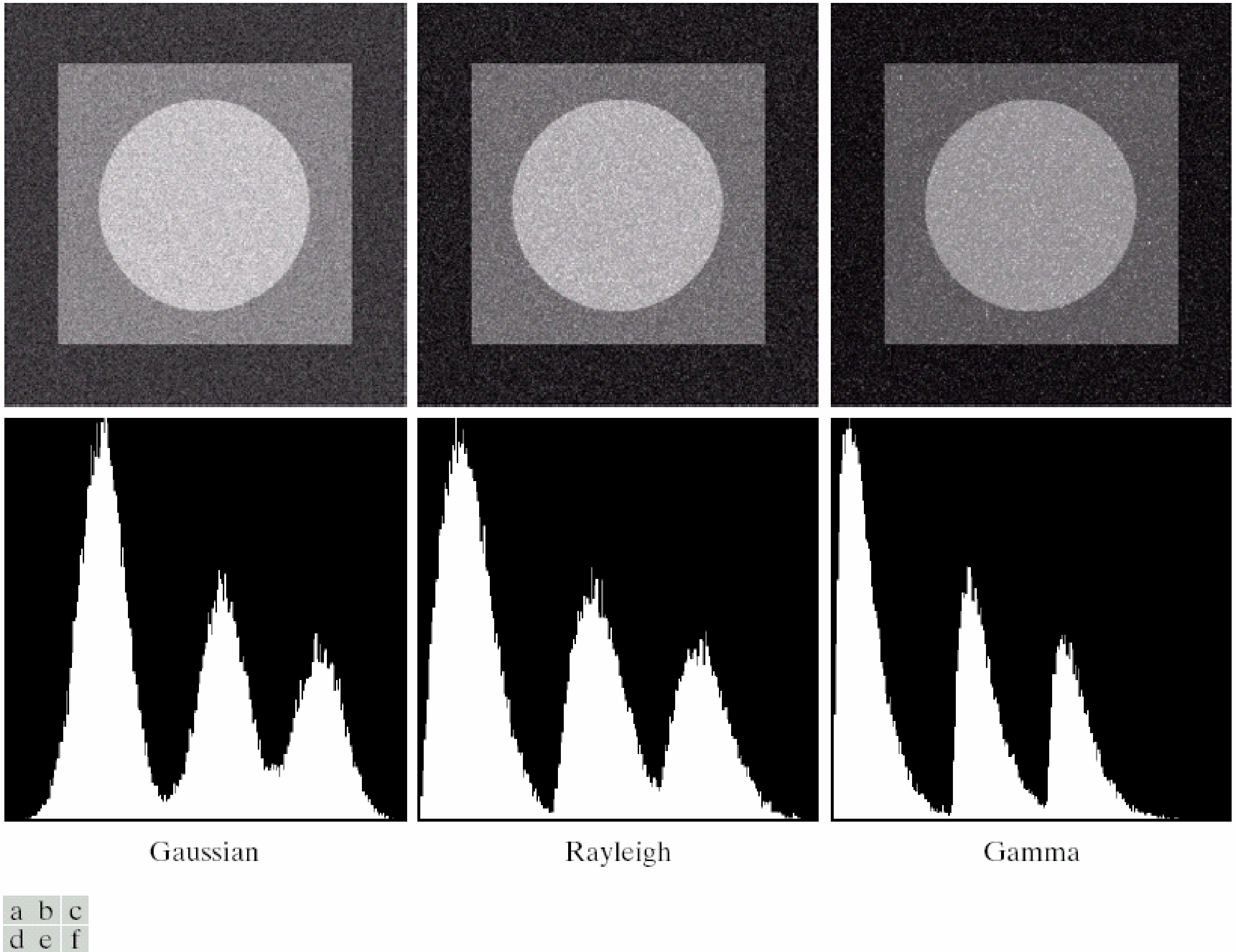

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

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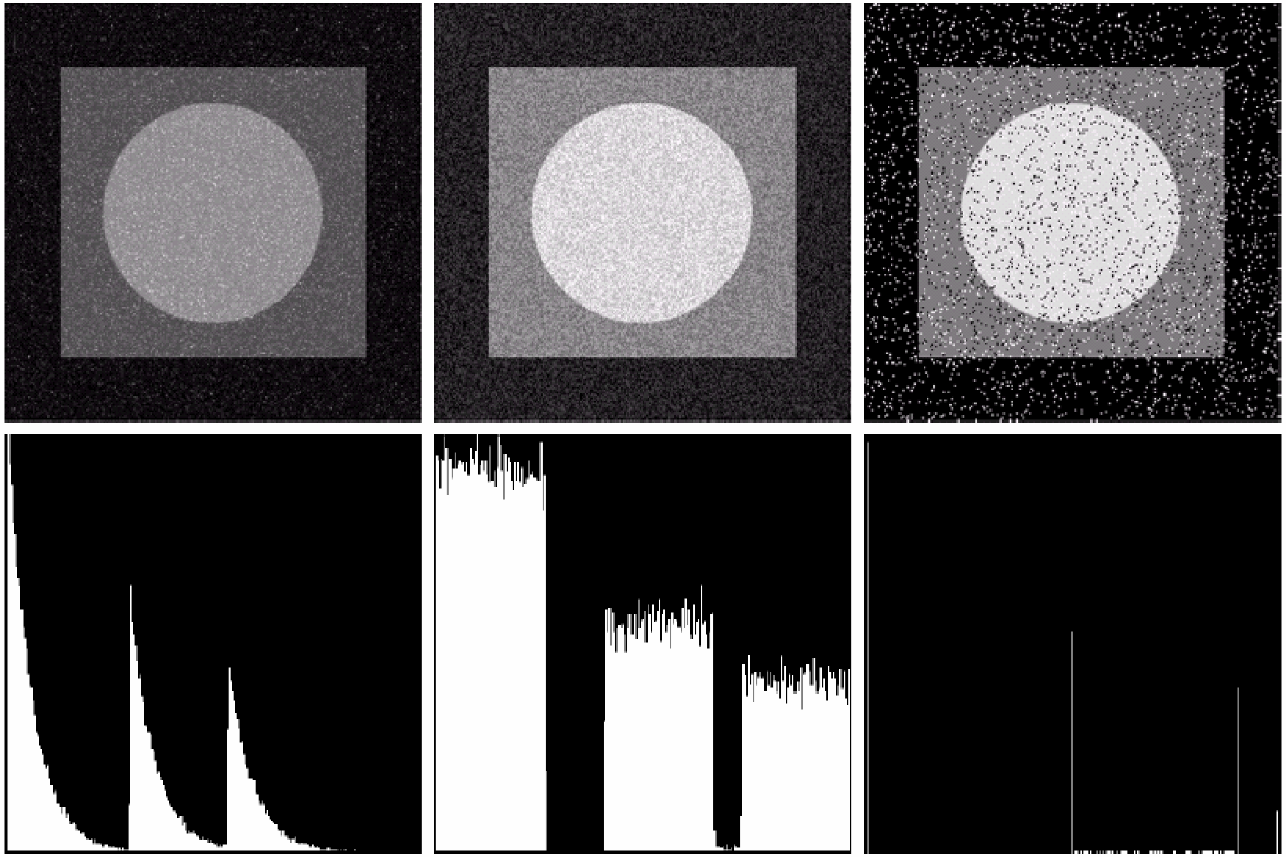

Exponential Uniform Salt & Pepper

g	h	i
j	k	l

FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and impulse noise to the image in Fig. 5.3.

$$\mu = \sum_{z_i \in S} z_i \, p(z_i) \qquad (5.2-15)$$

$$\sigma^2 = \sum_{z_i \in S} \left(z_i - \mu \right)^2 p(z_i) \qquad (5.2-16)$$

- If the shape is approximately Gaussian ,then the mean and variance is all we need because the Gaussian PDF is completely specified by these two parameters .
- For the other shapes discussed in Section 5.2.2,we use the mean and variance to solve for the parameters a and b.

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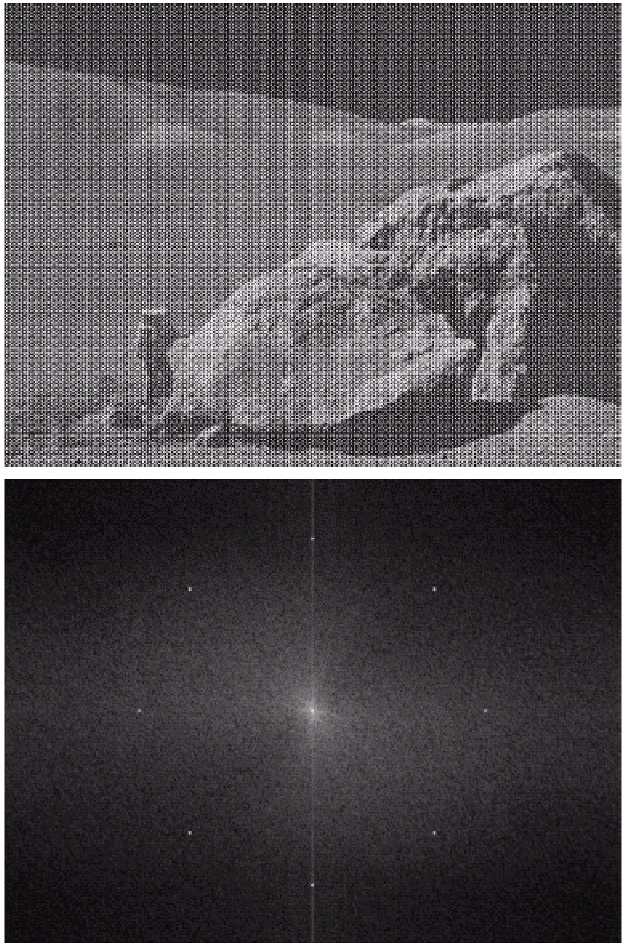

FIGURE 5.5 (a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

- 5.3 Restoration in the Presence of Noise Only- Spatial Filtering
- Spatial filtering is the method of choice in situations when only additive noise is present.

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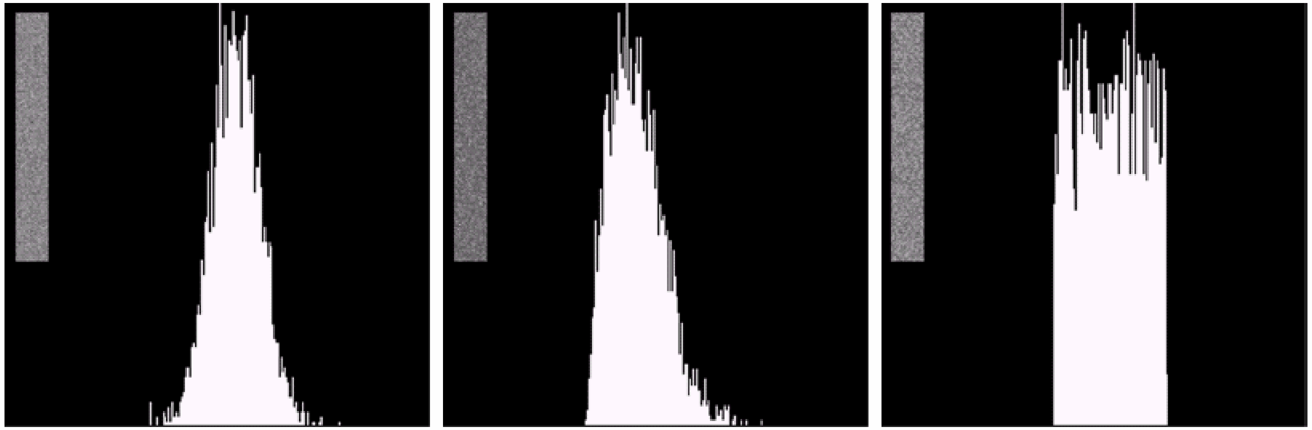


a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

- 5.3.1 Mean Filters
- Arithmetic mean filter

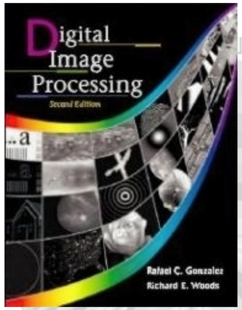
$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t) \qquad (5.3-3)$$

- A mean filter simply smoothes local variations in an image. Noise is reduced as a result of blurring.

- Geometric mean filter

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}} \qquad (5.3-4)$$

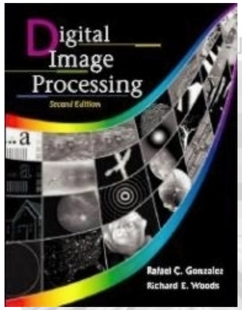
- A geometric mean filter achieves smoothing comparable to the arithmetic mean filter, but it tends to lose less image detail in the process.



- Harmonic mean filter

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}} \qquad (5.3-5)$$

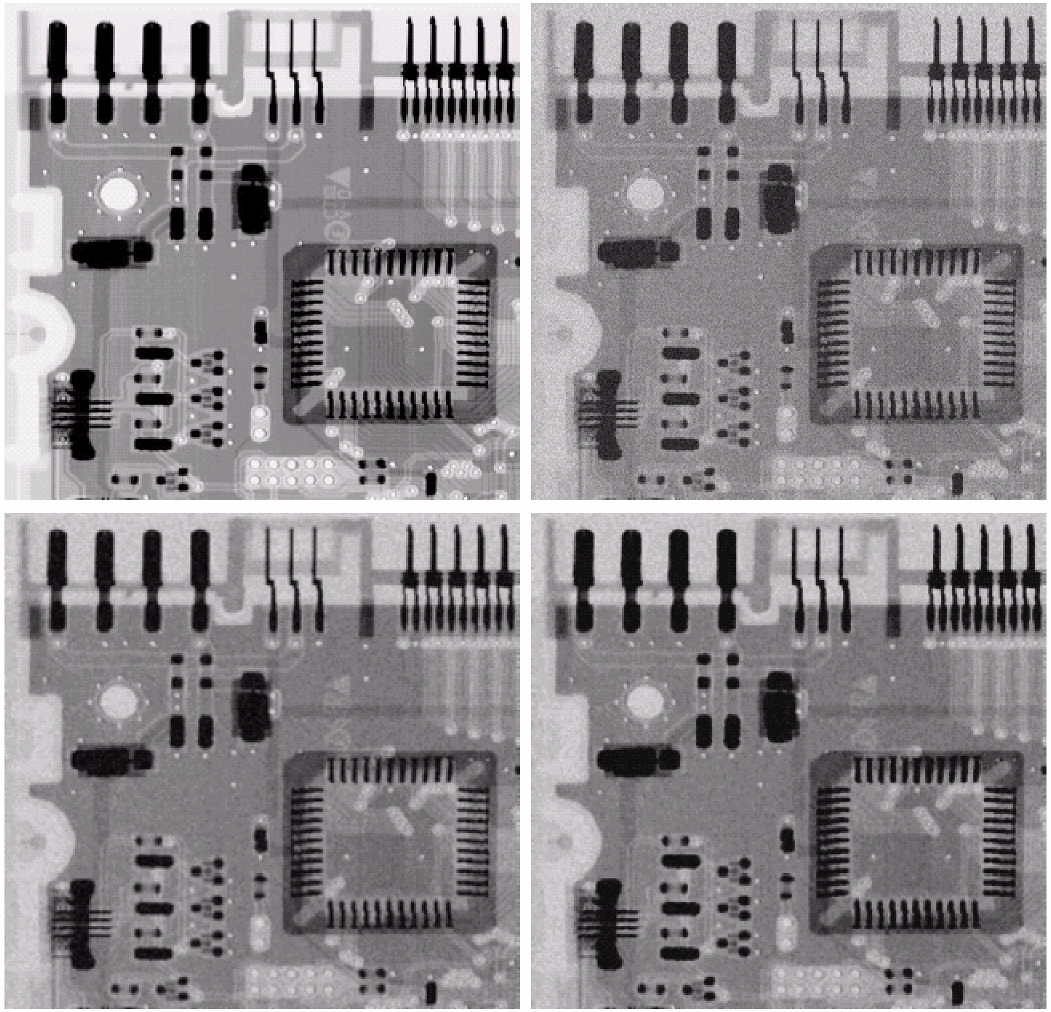
- The harmonic mean filter works well for salt noise, but fails for pepper noise. It does well also with other types of noise like Gaussian noise.



- Contraharmonic mean filter

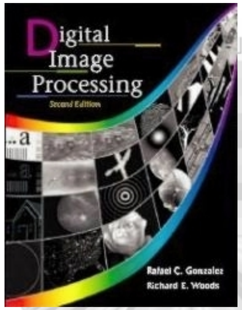
$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}} \qquad (5.3-6)$$

- where Q is called the order of the filter. This filter is well suited for reducing or virtually eliminating the effects of salt-and-pepper noise.
- For positive values of Q, the filter eliminates pepper noise. For negative values of Q it eliminates salt noise. It cannot do both simultaneously.



a b
c d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



- 5.3.2 Order-Statistics Filters
- Median filter

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\} \qquad (5.3-7)$$

- Certain types of random noise, they provide excellent noise-reduction capabilities, with considerably less blurring than linear smoothing filters .
- Median filters are particularly effective in the presence of both bipolar and unipolar impulse noise.

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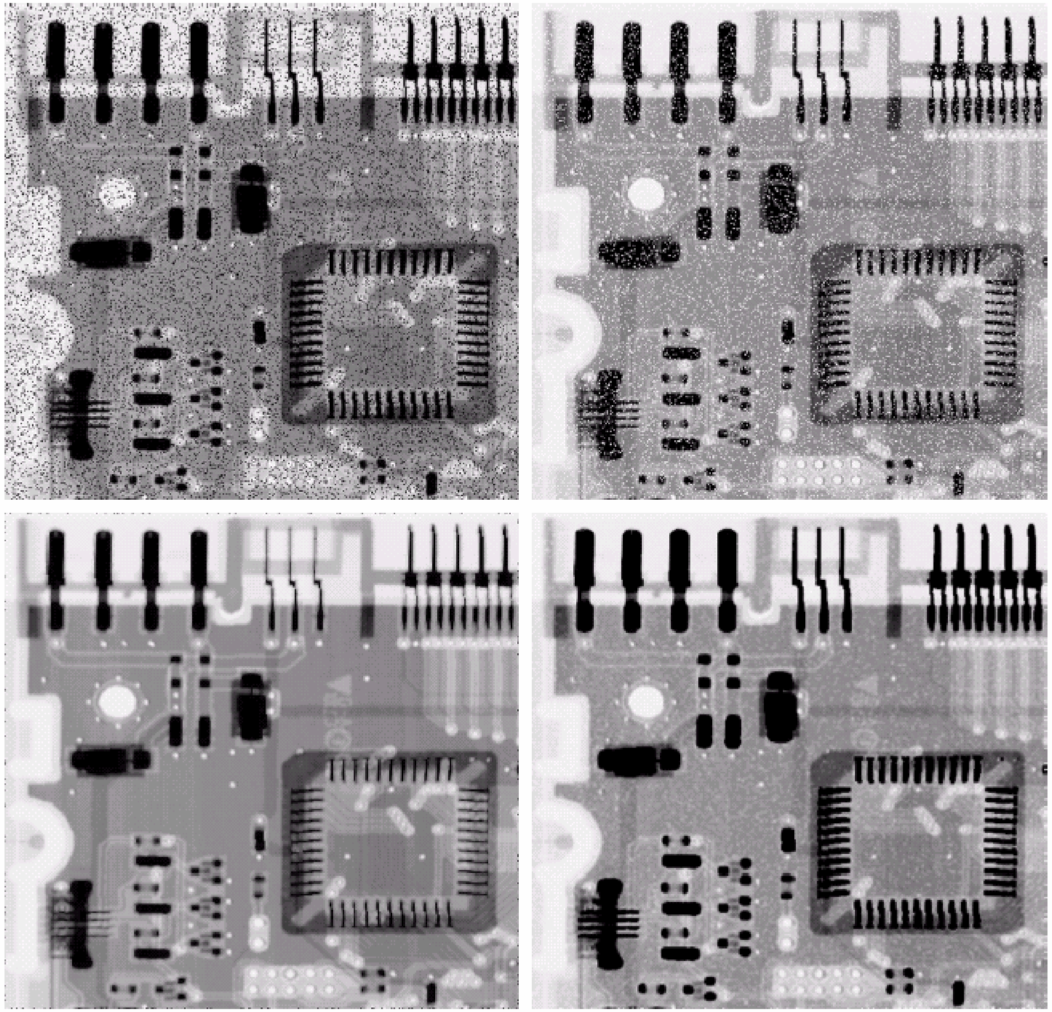

a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

- Max and min filters

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\} \qquad (5.3-8)$$

- This filter is useful for finding the brightest points in an image.

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\} \qquad (5.3-9)$$

- This filter is useful for finding the darkest points in an image.

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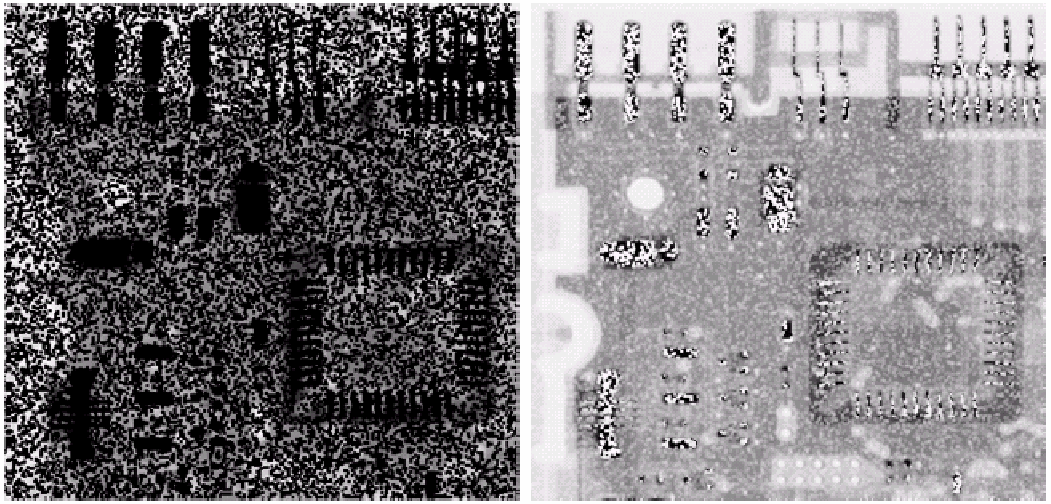

a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

- **Midpoint filter**

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right] \qquad (5.3-10)$$

- This filter works best for randomly distributed noise, like Gaussian or uniform noise.

- **Alpha-trimmed mean filter**
- Suppose that we delete the d/2 lowest and the d/2 highest gray-level values of $g(s,t)$ in the neighborhood S_{xy}.
- A filter formed by averaging these remaining pixels is called an alpha-trimmed mean filter:

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t) \qquad (5.3-11)$$

- The alpha-trimmed filter is useful in situations involving multiple types of noise, such as a combination of salt-and-pepper and Gaussian noise.

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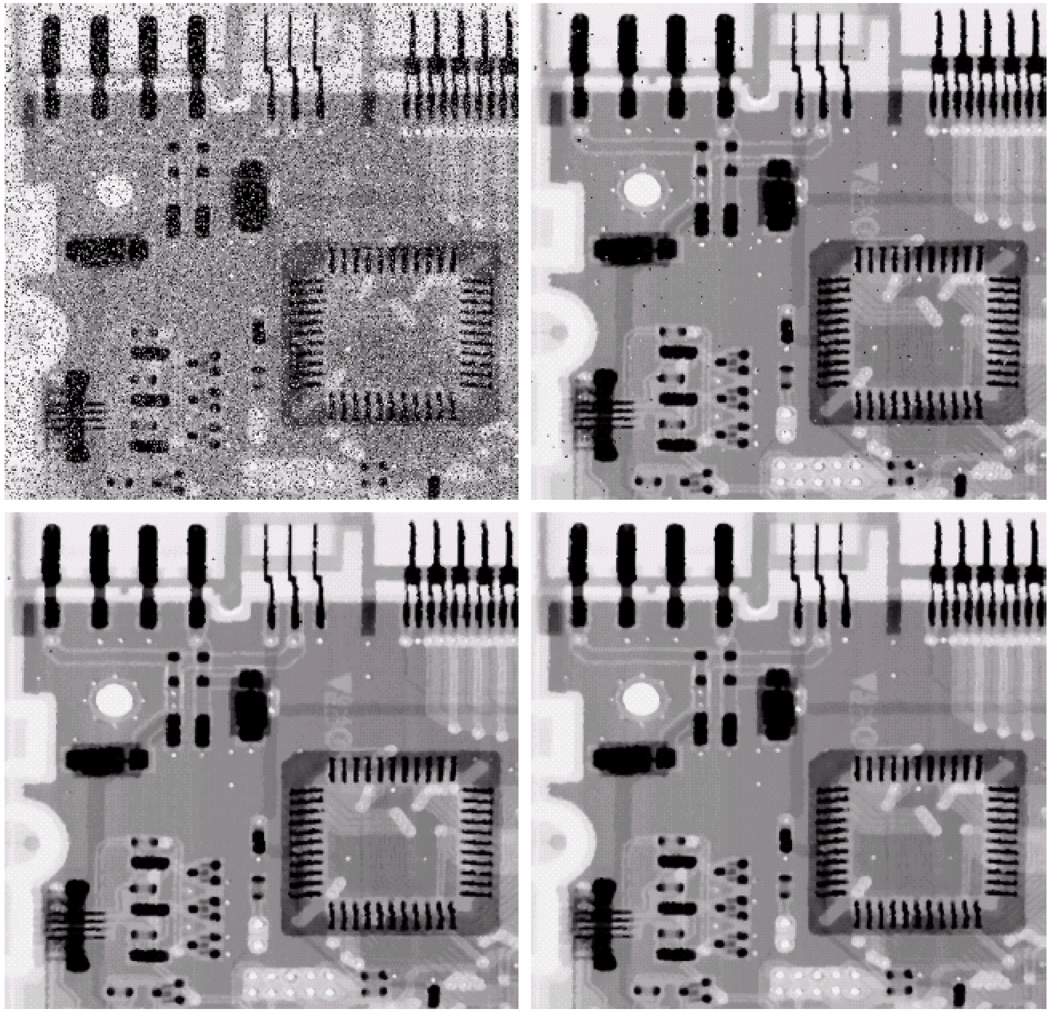

a b
c d

FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.

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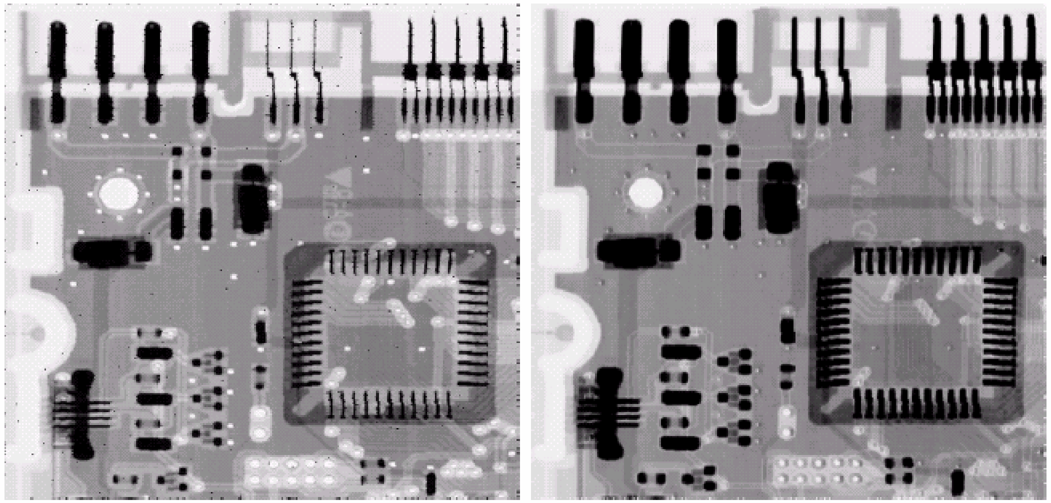
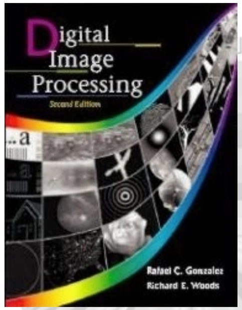


a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3 × 3. (b) Result of filtering 5.8(b) with a min filter of the same size.



5.3.3 Adaptive Filters

- **Adaptive local noise reduction filter**

1. If σ_η^2 is zero, the filter should return simply the value of g(x,y). This is the trivial, zero-noise case in which g(x,y) is equal to f(x,y).

2. If the local variance is high relative to σ_η^2, the filter should return a value close to g(x. y). A high local variance typically is associated with edges, and these should be preserved.

3. If the two variances are equal, we want the filter to return the arithmetic mean value of the pixels in S_{xy}. This condition occurs when the local area has the same properties as the overall image, and local noise is to be reduced simply by averaging

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_{\eta}^{2}}{\sigma_{L}^{2}}\left[g(x,y) - m_{L}\right] \qquad (5.3-12)$$

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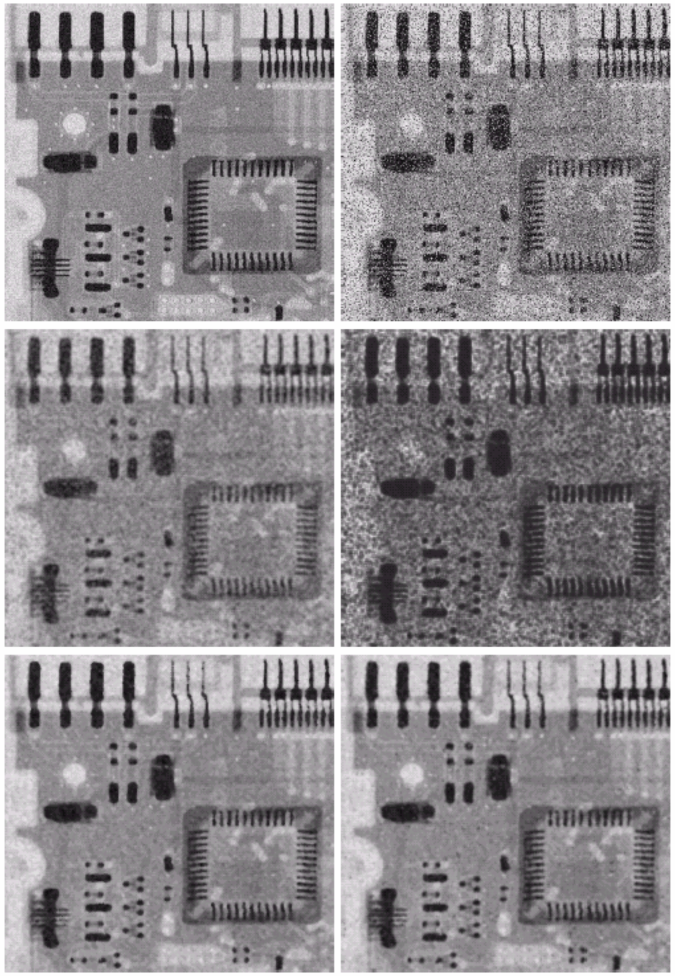

a b
c d
e f

FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image in (b) filtered with a 5×5: (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.

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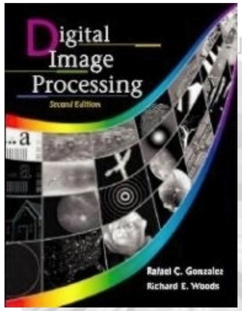
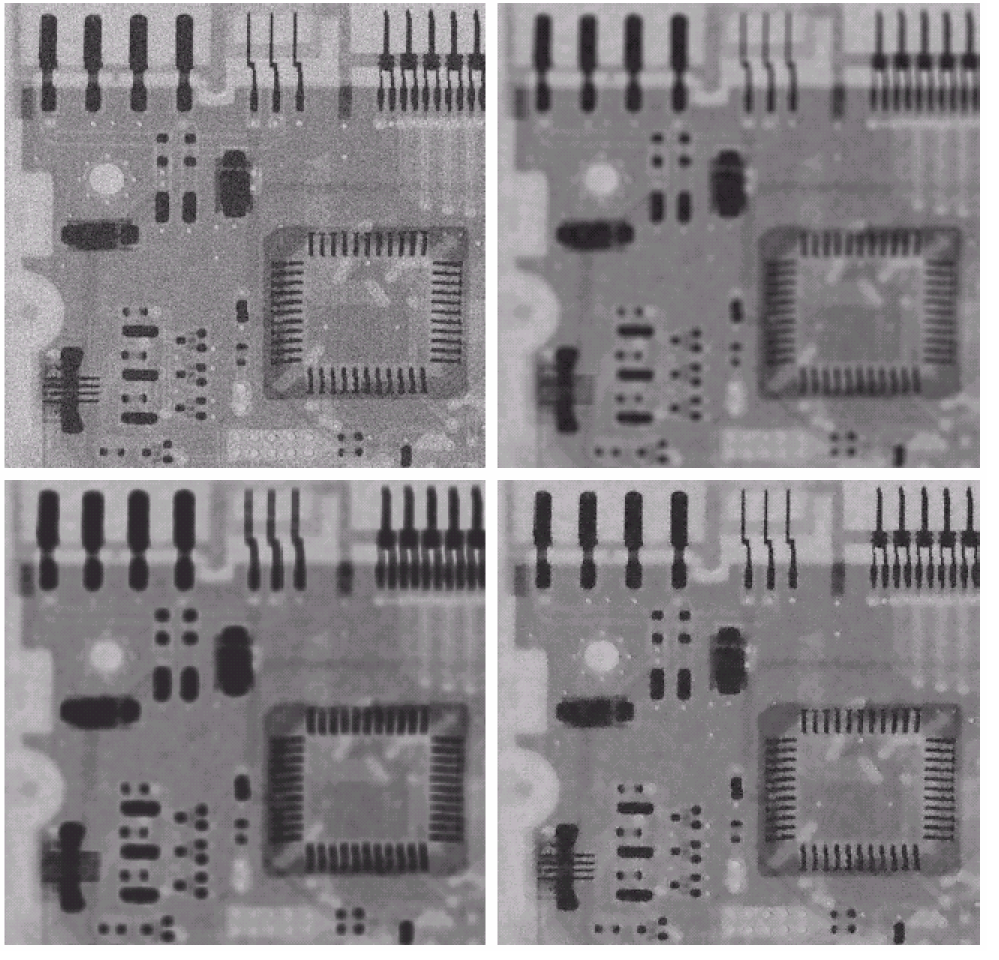
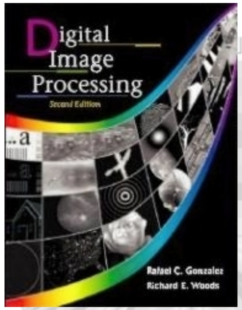


a b
c d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7×7.





- **Adaptive median filter**
- z_{min} = minimum gray level value in S_{xy}
- z_{max} = maximum gray level value in S_{xy}
- z_{med} = median of gray levels in S_{xy}
- z_{xy} = gray level at coordinates (x, y)
- S_{max} = maximum allowed size of S_{xy}.

- Level A:

$$A1 = z_{med} - z_{\min}$$

$$A2 = z_{med} - z_{\max}$$

If Al > 0 AND A2 < 0, Go to level B

Else increase the window size

If window $size \le S_{\max}$ repeat level A

Else output Z_{xy}

- Level B:

$$B1 = z_{xy} - z_{\min}$$

$$B2 = z_{xy} - z_{\max}$$

If Bl > 0 AND B2 < 0, output z_{xy}

Else output Z_{med}

- The key to understanding the mechanics of this algorithm is to keep in mind that it has three main purposes: to remove salt-and-pepper (impulse) noise, to provide smoothing of other noise that may not be impulsive, and to reduce distortion, such as excessive thinning or thickening of object boundaries.

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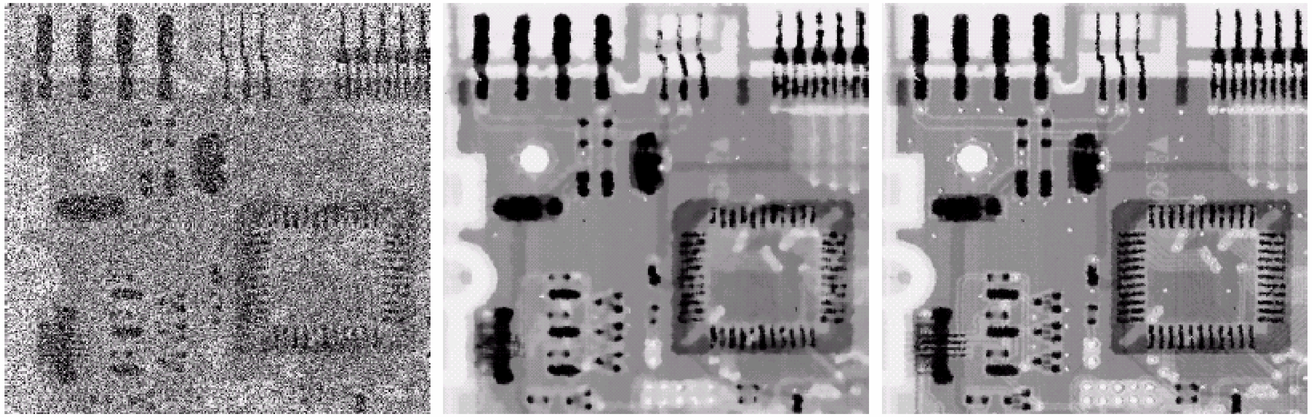
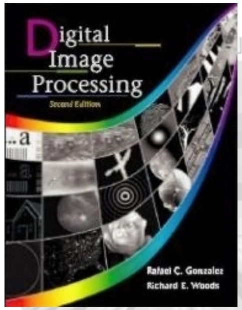


a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.



5.4 Periodic Noise Reduction by Frequency Domain Filtering

- 5.4.1 Bandreject Filters
- Bandreject filters remove or attenuate a band of frequencies about the origin of the Fourier transform.

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) < D_0 - \dfrac{w}{2} \\ 0 & \text{if } D_0 - \dfrac{w}{2} \le D(u,v) < D_0 + \dfrac{w}{2} \\ 1 & \text{if } D(u,v) > D_0 - \dfrac{w}{2} \end{cases} \qquad (5.4-1)$$

- where D (u, v) is the distance from the origin of the centered frequency rectangle, as given in Eq.(4.3-3), W is the width of the band, and D_0 is its radial center.

- A Butterworth bandreject filter of order n

$$H(u,v) = \frac{1}{1 + \left[\frac{D(u,v)w}{D^2(u,v) - D_0^2} \right]} \qquad (5.4-2)$$

- Gaussian bandreject filter is given by

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D^2(u,v) - D_0^2}{D(u,v)w} \right]} \qquad (5.4-3)$$

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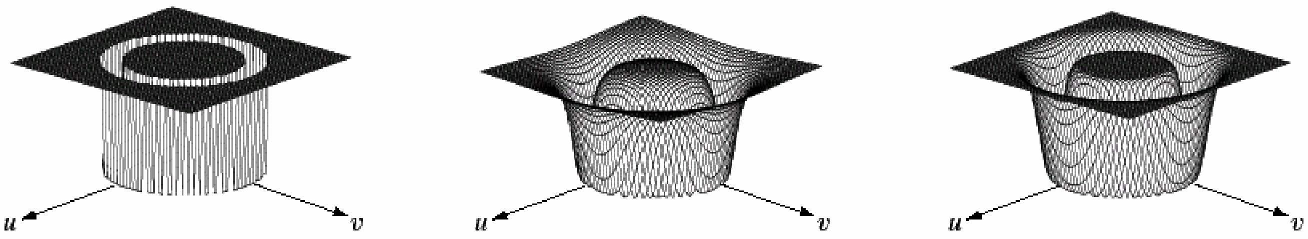

FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

- A good example is an image corrupted by additive periodic noise that can be approximated as two-dimensional sinusoidal functions.

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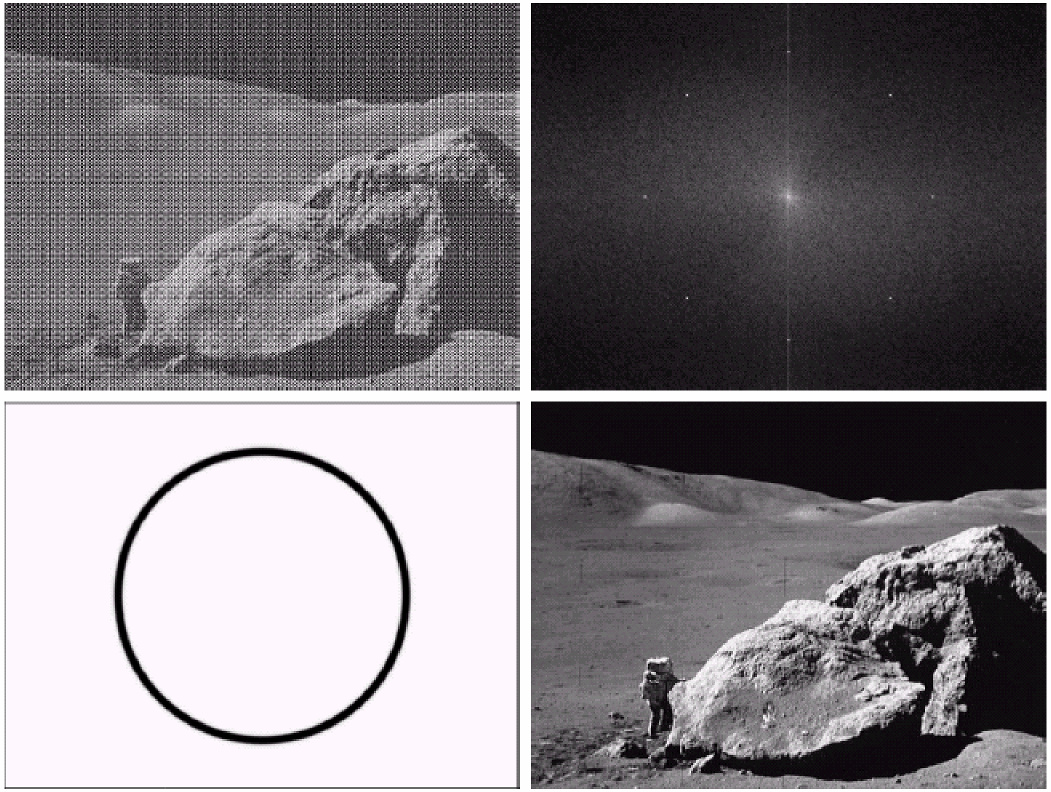

a b
c d

FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

- 5.4.2 Bandpass Filters

$$H_{bp}(u,v) = 1 - H_{br}(u,v) \qquad (5.4-4)$$

- 5.4.3 Notch Filters
- A notch filter rejects (or passes) frequencies in predefined neighborhoods about a center frequency. Figure 5.18 shows 3-D plots of ideal, Butterworth, an Gaussian notch (reject) filters.

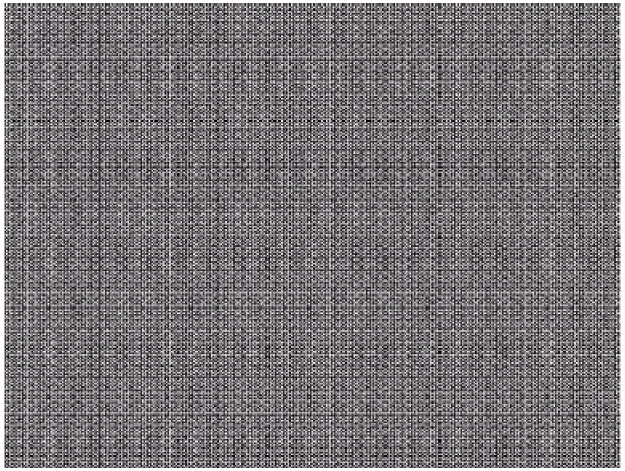

FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.

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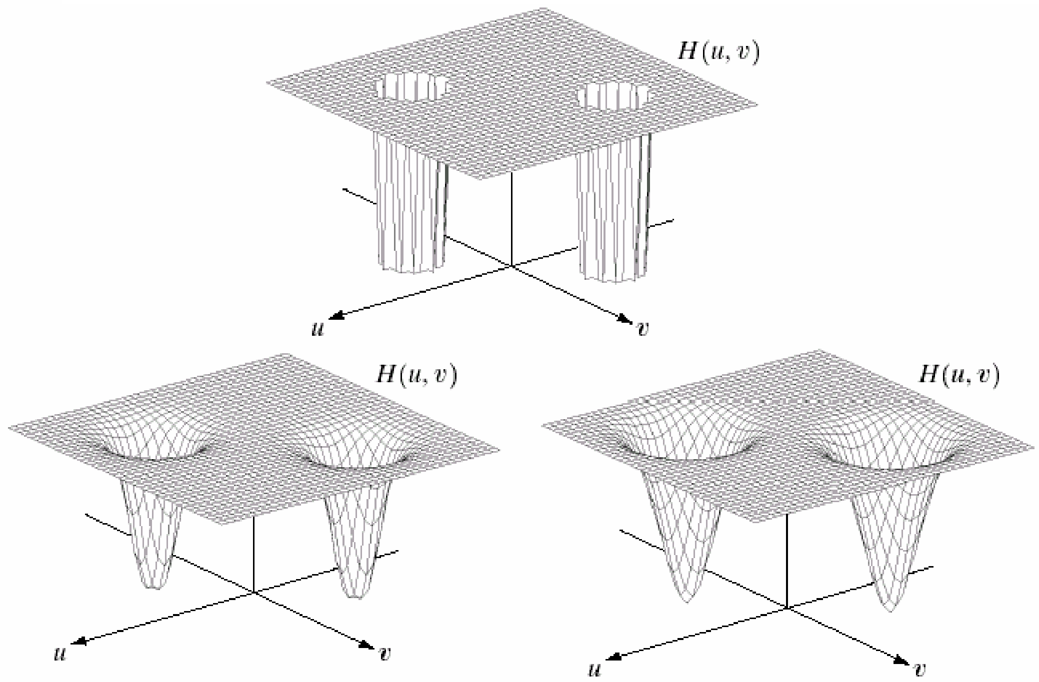

FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.

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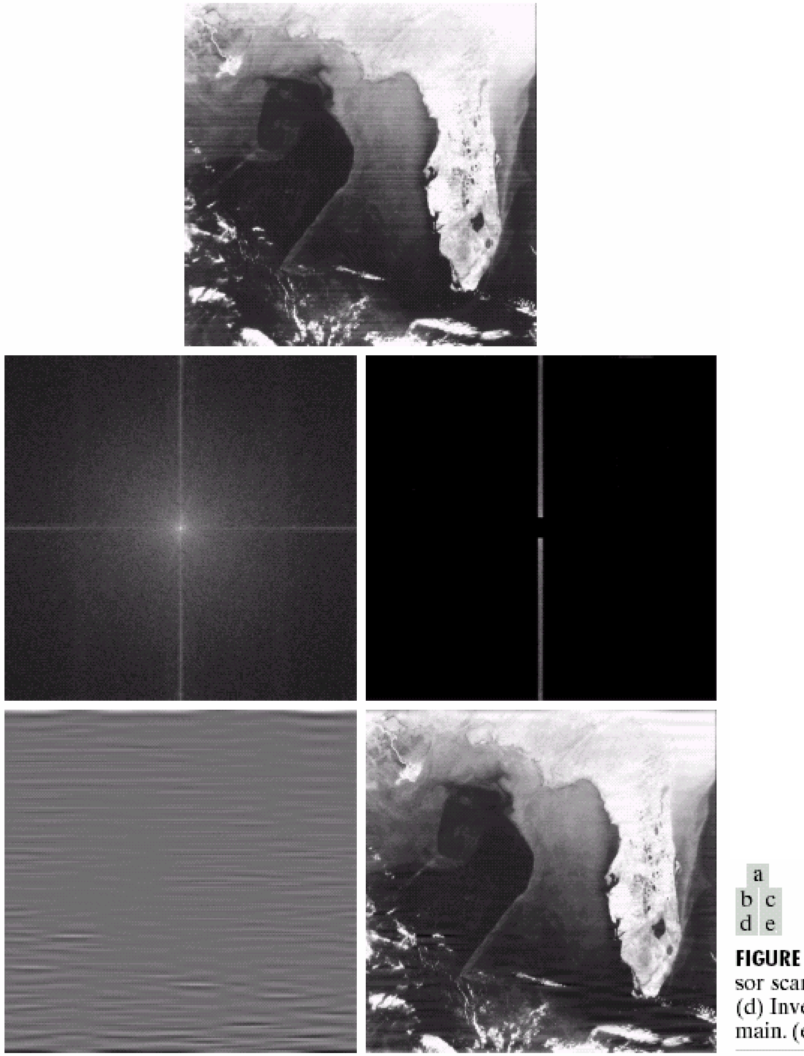

FIGURE 5.19 (a) Satellite image of Florida and the Gulf of Mexico (note horizontal sensor scan lines). (b) Spectrum of (a). (c) Notch pass filter shown superimposed on (b). (d) Inverse Fourier transform of filtered image, showing noise pattern in the spatial domain. (e) Result of notch reject filtering. (Original image courtesy of NOAA.)

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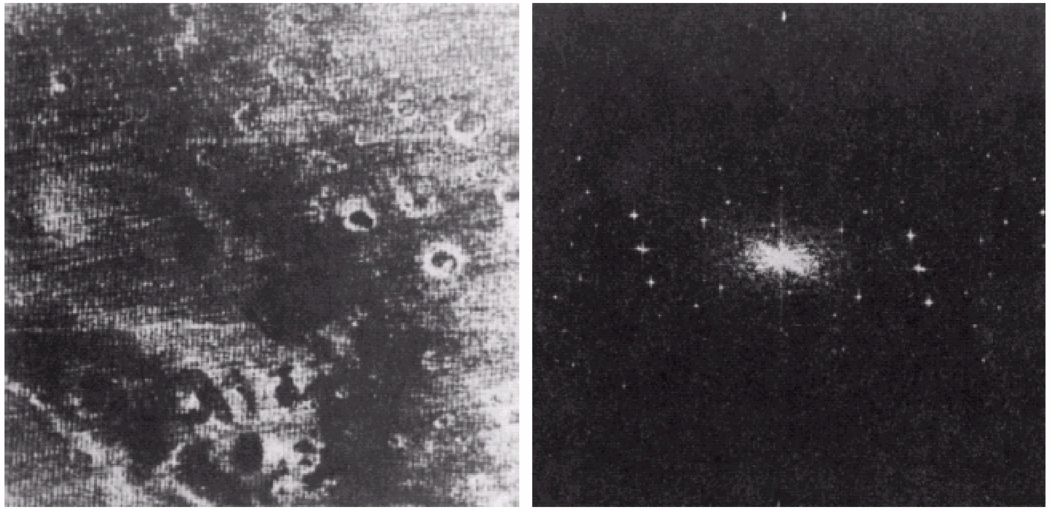

a b

FIGURE 5.20
(a) Image of the Martian terrain taken by *Mariner 6*. (b) Fourier spectrum showing periodic interference. (Courtesy of NASA.)

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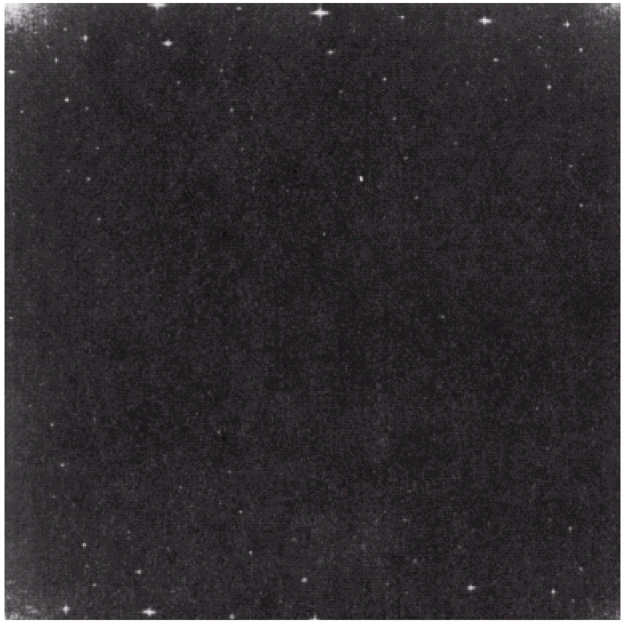

FIGURE 5.21 Fourier spectrum (without shifting) of the image shown in Fig. 5.20(a). (Courtesy of NASA.)

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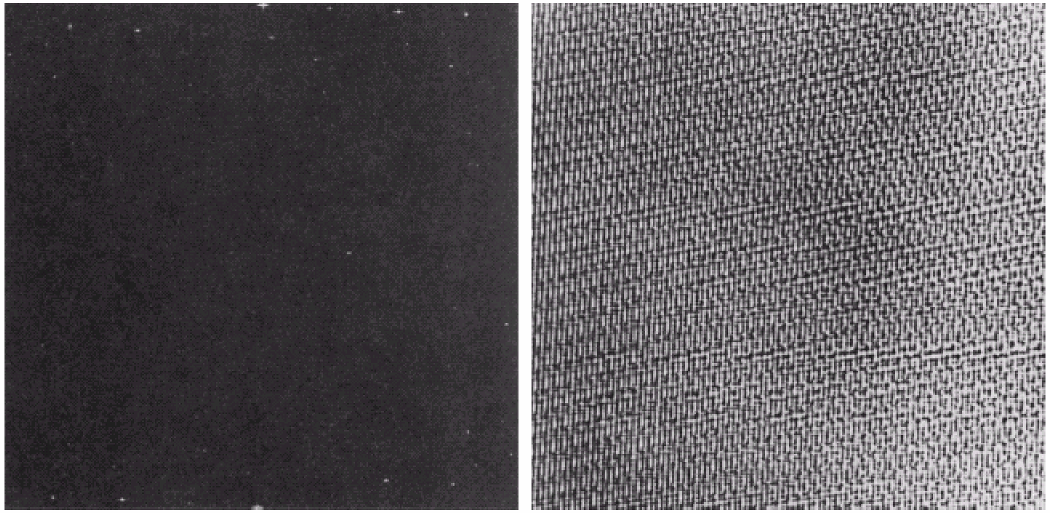

a b

FIGURE 5.22 (a) Fourier spectrum of $N(u, v)$, and (b) corresponding noise interference pattern $\eta(x, y)$. (Courtesy of NASA.)

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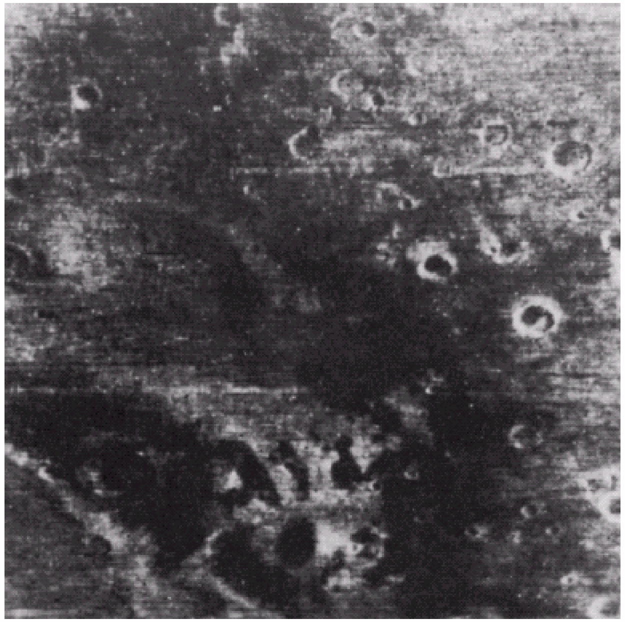

FIGURE 5.23 Processed image. (Courtesy of NASA.)

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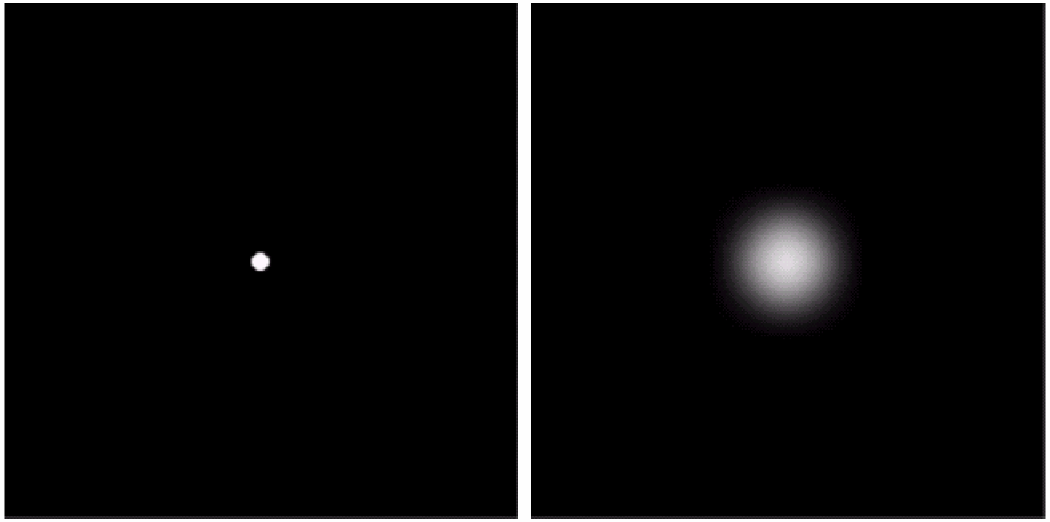

a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

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a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

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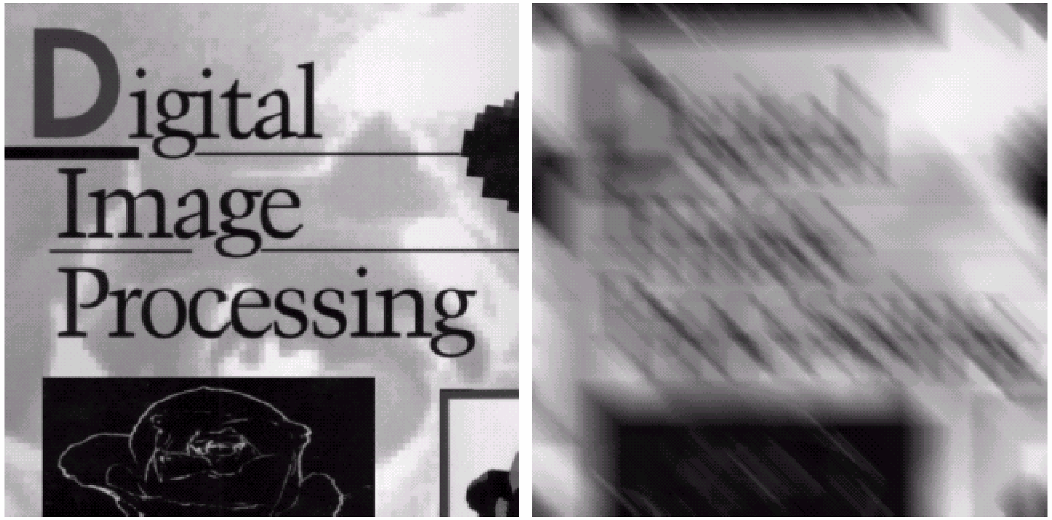

a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

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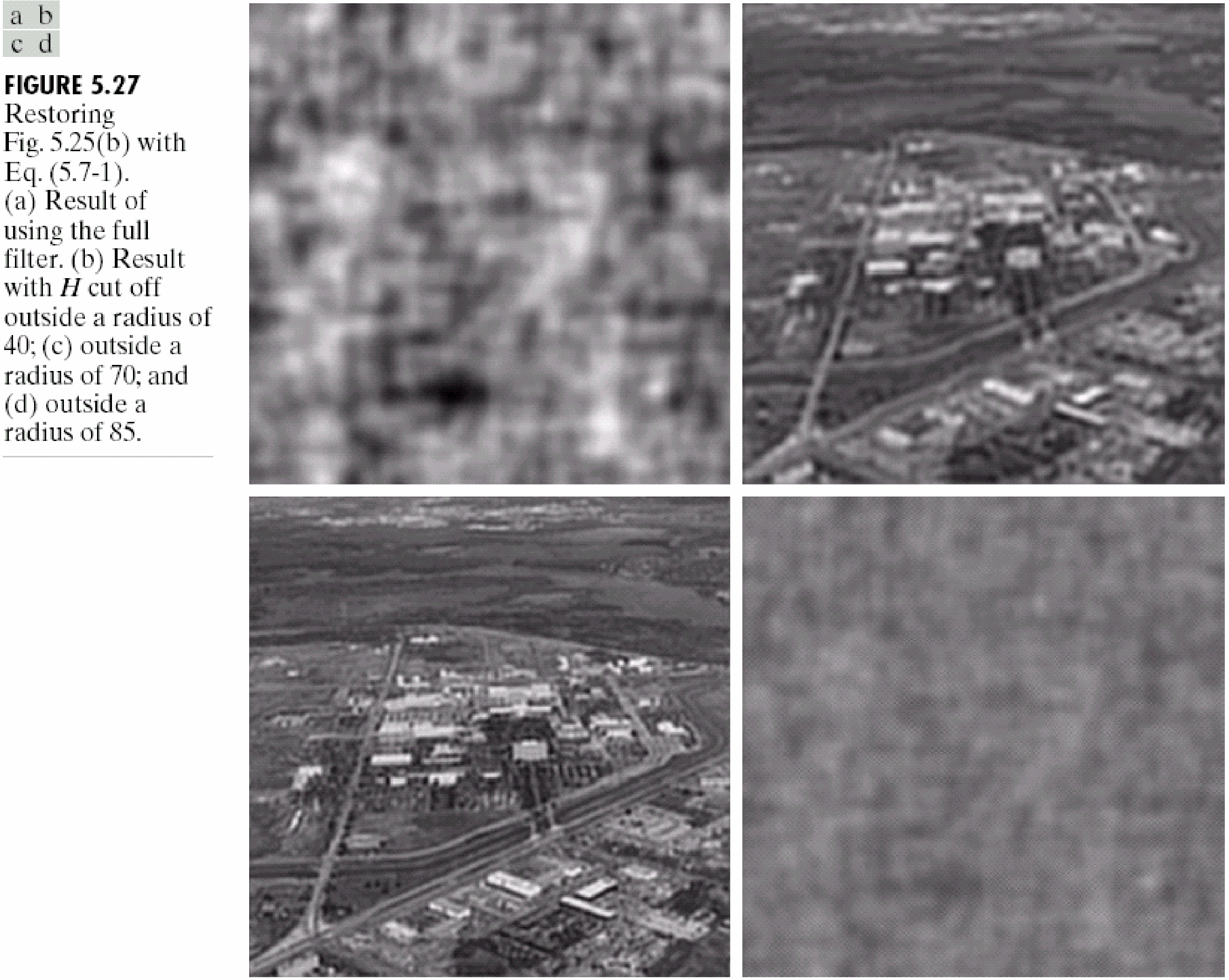

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with *H* cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

5.8 Minimum Mean Square Error (Wiener) Filtering

$$e^2 = E\left\{\left(f - \hat{f}\right)^2\right\} \qquad (5.8-1)$$

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)\left|H(u,v)\right|^2 + S_\eta(u,v)}\right]G(u,v)$$

$$= \left[\frac{H^*(u,v)}{\left|H(u,v)\right|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v) \qquad (5.8-2)$$

$$= \left[\frac{1}{H(u,v)}\frac{\left|H(u,v)\right|^2}{\left|H(u,v)\right|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)$$

$$\hat{F}(u,v)=\left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2+k}\right]G(u,v) \qquad (5.8-3)$$

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a b c

FIGURE 5.28 Comparison of inverse- and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

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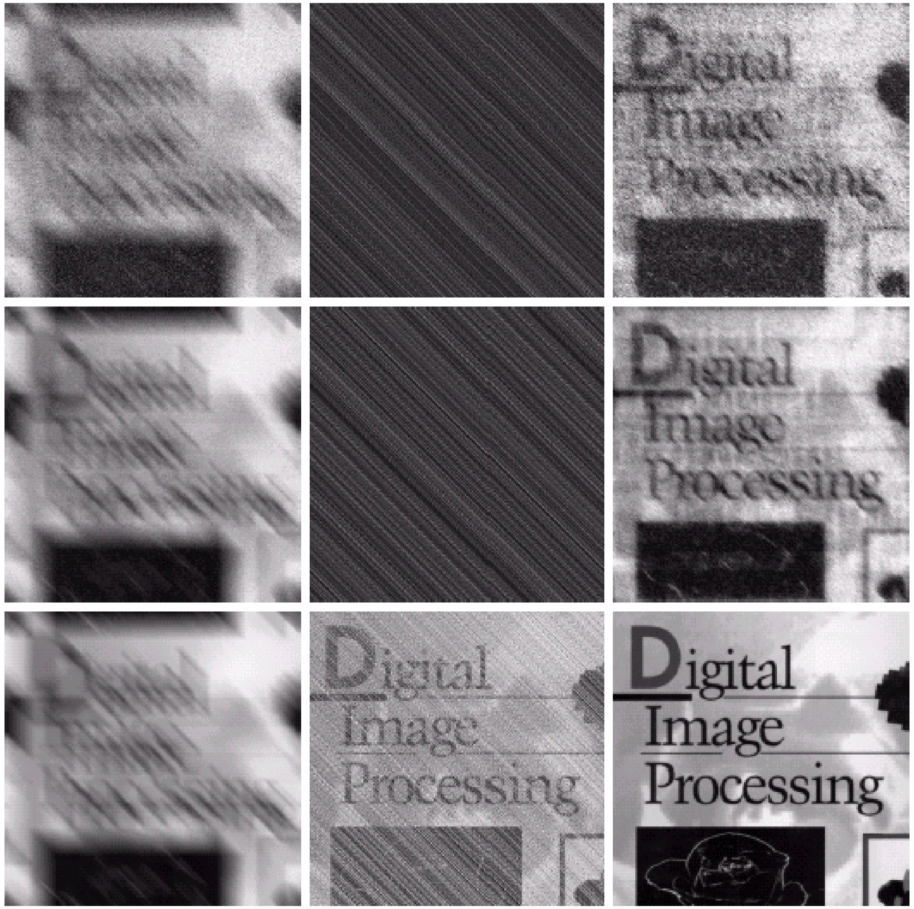

a b c
d e f
g h i

FIGURE 5.29 (a) Image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a "curtain" of noise.

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a b c

FIGURE 5.30 Results of constrained least squares filtering. Compare (a), (b), and (c) with the Wiener filtering results in Figs. 5.29(c), (f), and (i), respectively.

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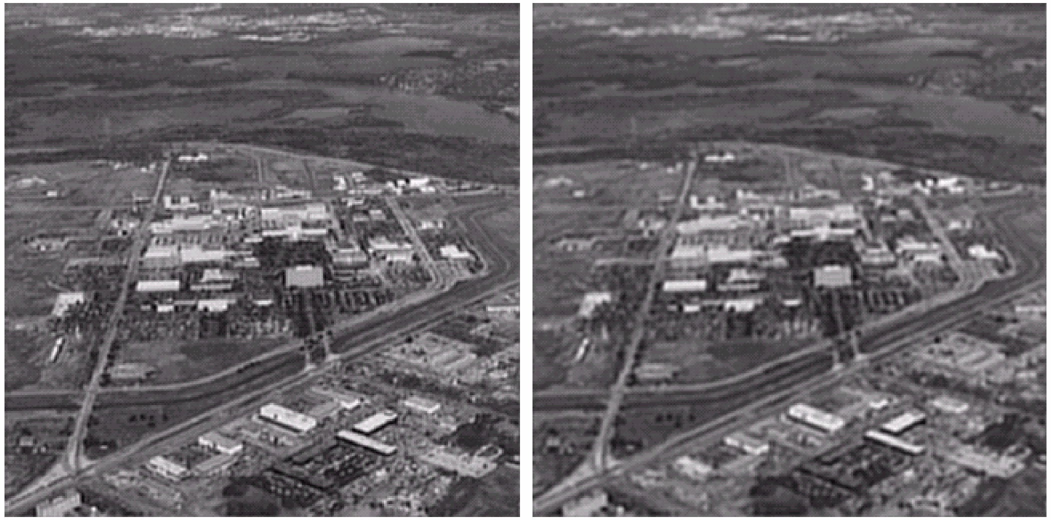

a b

FIGURE 5.31 (a) Iteratively determined constrained least squares restoration of Fig. 5.16(b), using correct noise parameters. (b) Result obtained with wrong noise parameters.

5.11 Geometric Transformations

- **5.11.1 Spatial Transformations**

$$x' = r(x, y) \qquad (5.11-1)$$

$$y' = s(x, y) \qquad (5.11-2)$$

$$r(x,y) = c_1 x + c_2 y + c_3 xy + c_4 \qquad (5.11-3)$$

$$s(x,y) = c_5 x + c_6 y + c_7 xy + c_8 \qquad (5.11-4)$$

$$x' = c_1 x + c_2 y + c_3 xy + c_4 \qquad (5.11-5)$$

$$y' = c_5 x + c_6 y + c_7 xy + c_8 \qquad (5.11-6)$$

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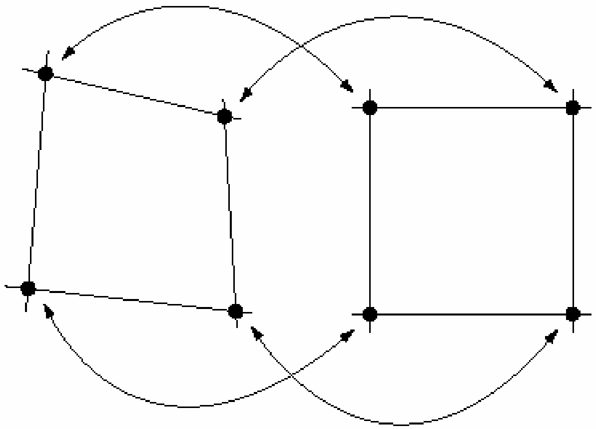

FIGURE 5.32 Corresponding tiepoints in two image segments.

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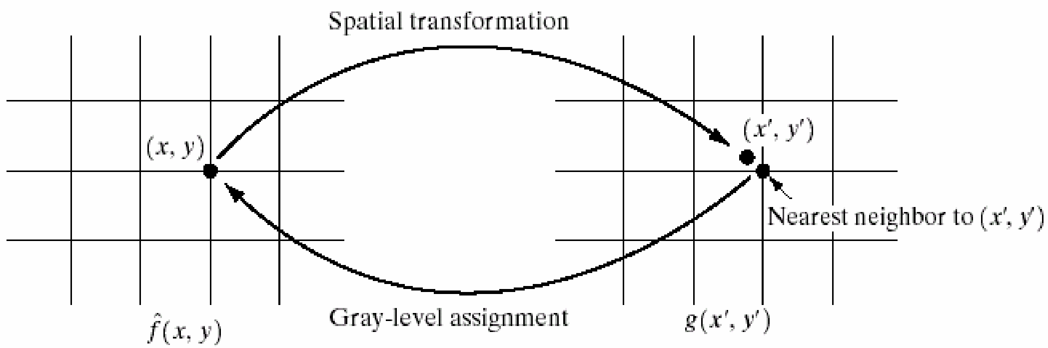

FIGURE 5.33 Gray-level interpolation based on the nearest neighbor concept.

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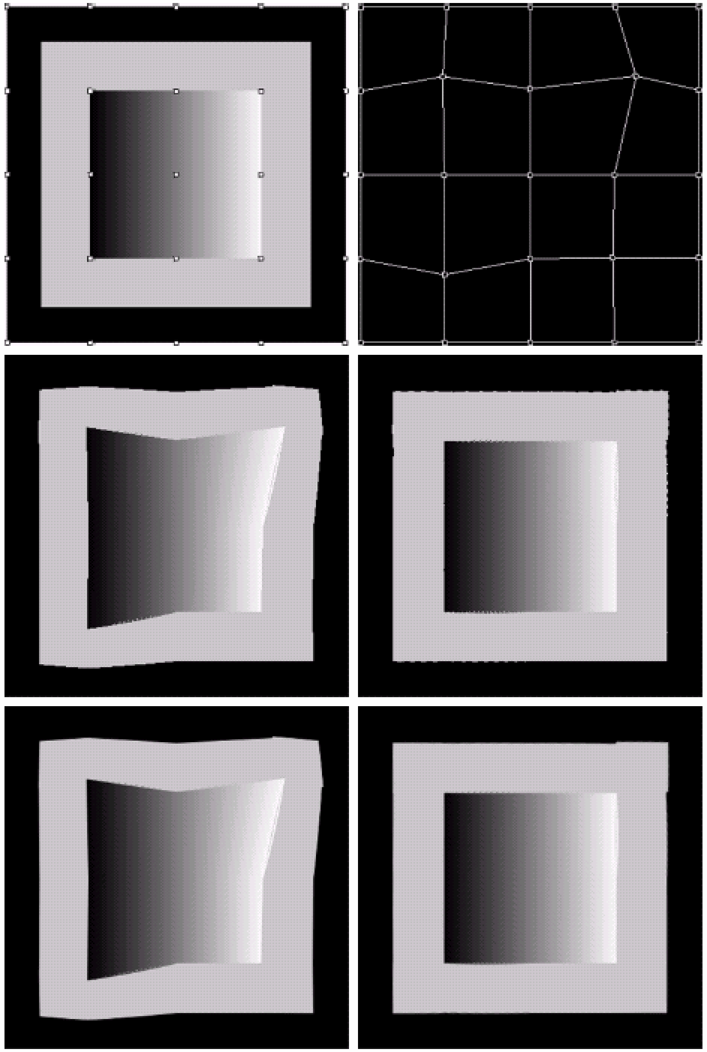

a b
c d
e f

FIGURE 5.34 (a) Image showing tiepoints. (b) Tiepoints after geometric distortion. (c) Geometrically distorted image, using nearest neighbor interpolation. (d) Restored result. (e) Image distorted using bilinear interpolation. (f) Restored image.

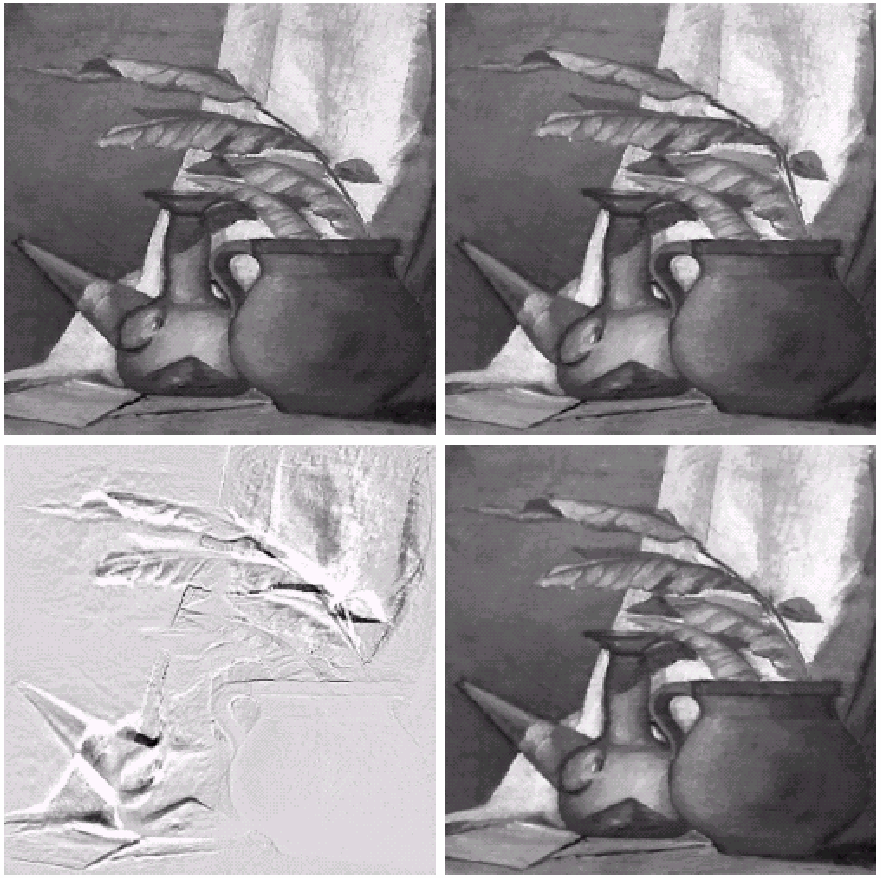

FIGURE 5.35 (a) An image before geometric distortion. (b) Image geometrically distorted using the same parameters as in Fig. 5.34(e). (c) Difference between (a) and (b). (d) Geometrically restored image.