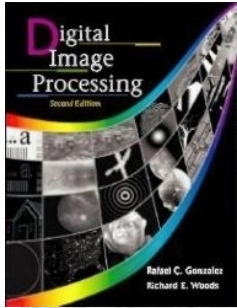


# Image Enhancement

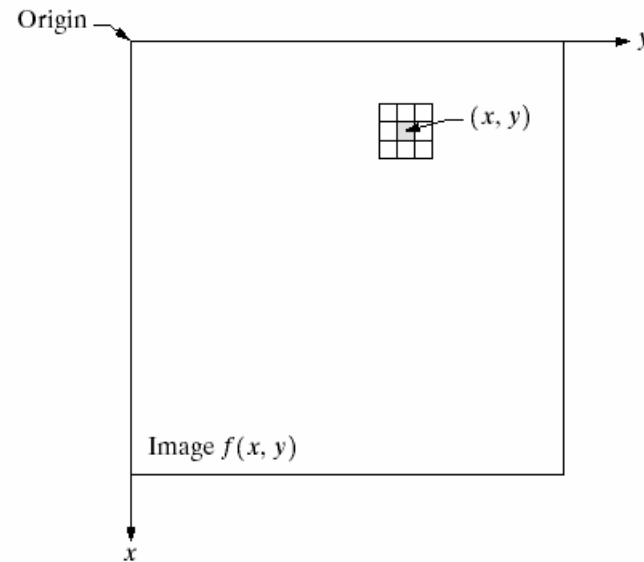
- Spatial Domain (第三章)
- Frequency Domain (第四章)

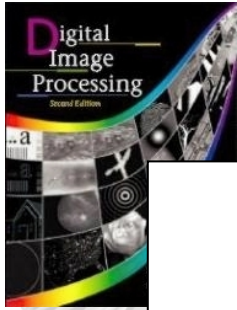


## Chapter 3

# Image Enhancement in the Spatial Domain

**FIGURE 3.1** A  $3 \times 3$  neighborhood about a point  $(x, y)$  in an image.



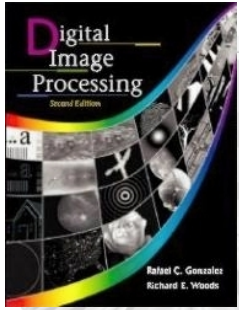


## 3.1 Transformation Function

$$g(x,y) = T [f(x,y)]$$

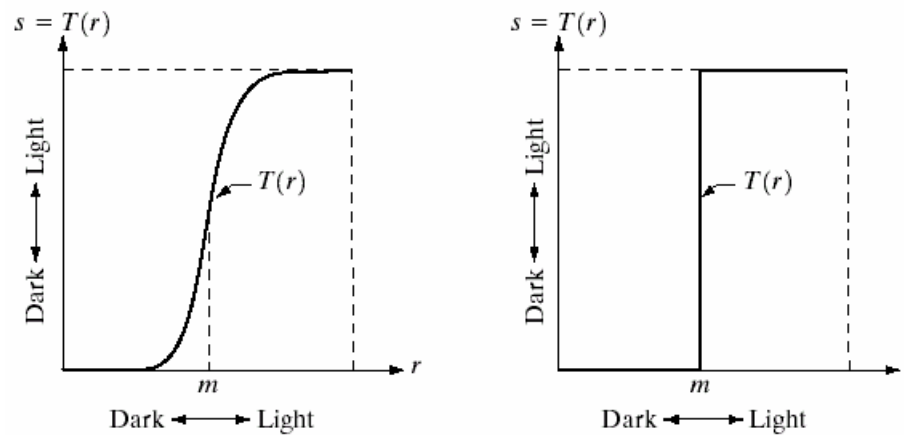
$f$  : original image

$g$  : enhanced image

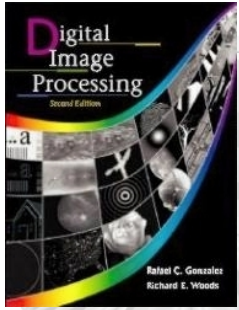


# Chapter 3

## Image Enhancement in the Spatial Domain

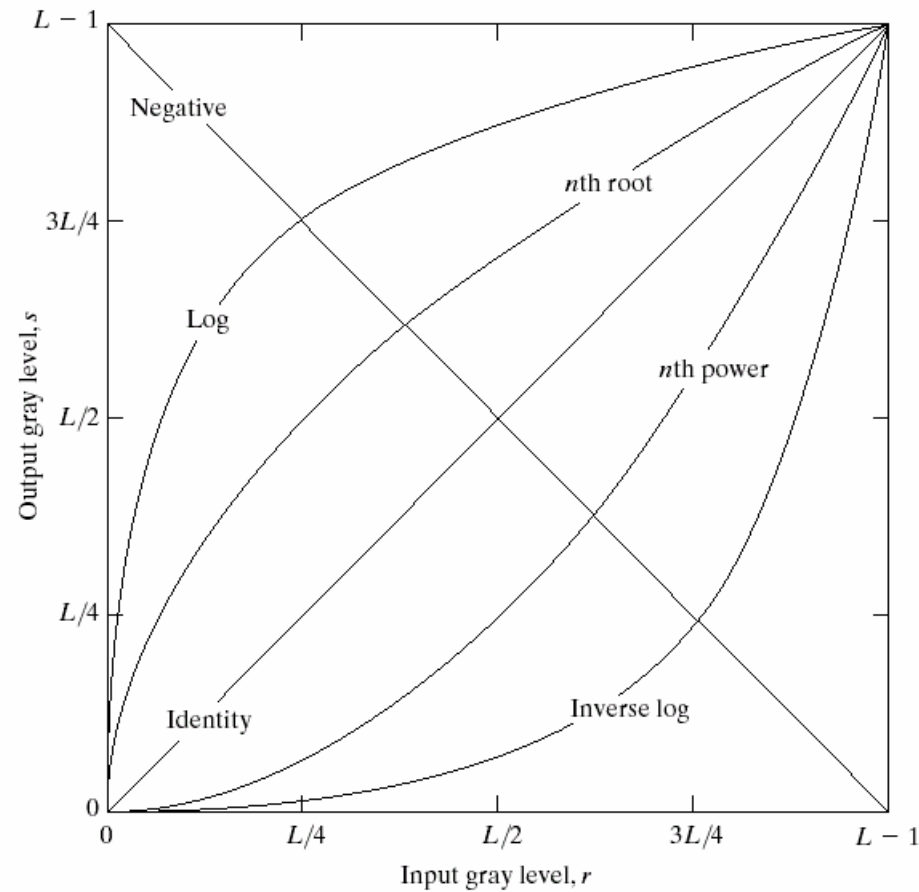


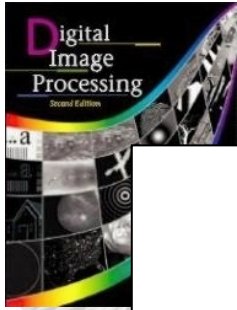
a b  
**FIGURE 3.2** Gray-level transformation functions for contrast enhancement.



## Chapter 3 Image Enhancement in the Spatial Domain

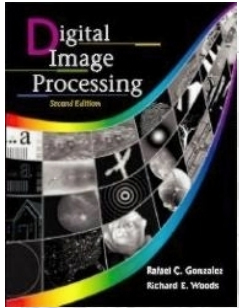
**FIGURE 3.3** Some basic gray-level transformation functions used for image enhancement.



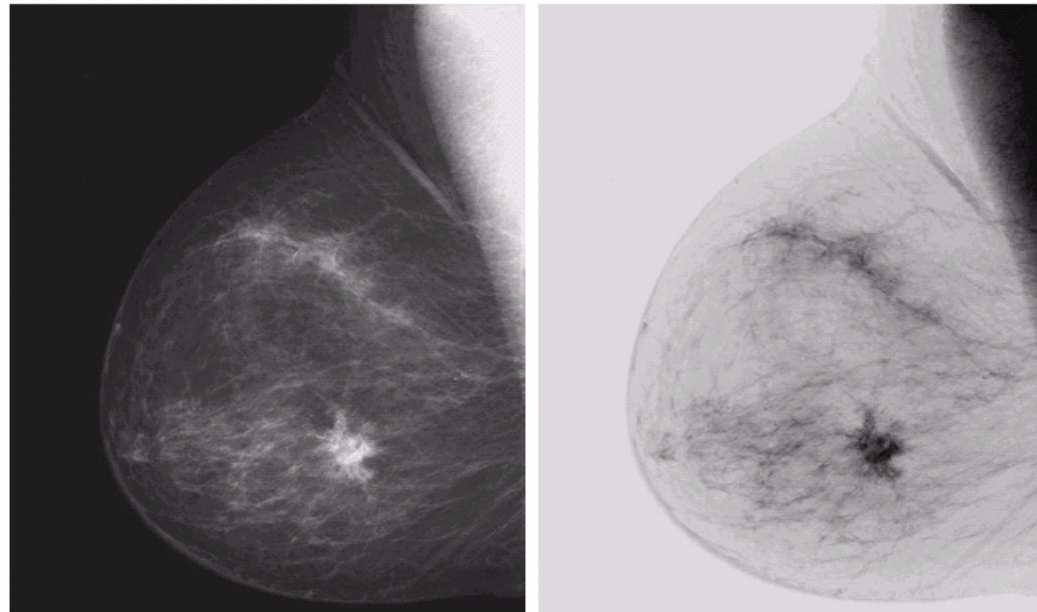


# Enhancement

- 1 . Negative : slope =  $-1$



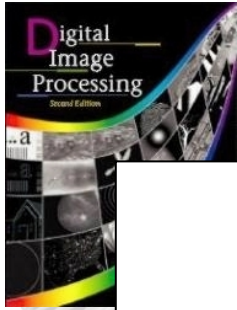
## Chapter 3 Image Enhancement in the Spatial Domain



a b

**FIGURE 3.4**

(a) Original digital mammogram.  
(b) Negative image obtained using the negative transformation in Eq. (3.2-1).  
(Courtesy of G.E. Medical Systems.)



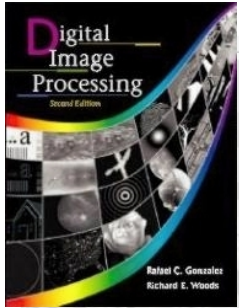
# Enhancement

- 2 . Log :  $s = c * \log(1+r)$

$T = \log$

$c$  : 常數



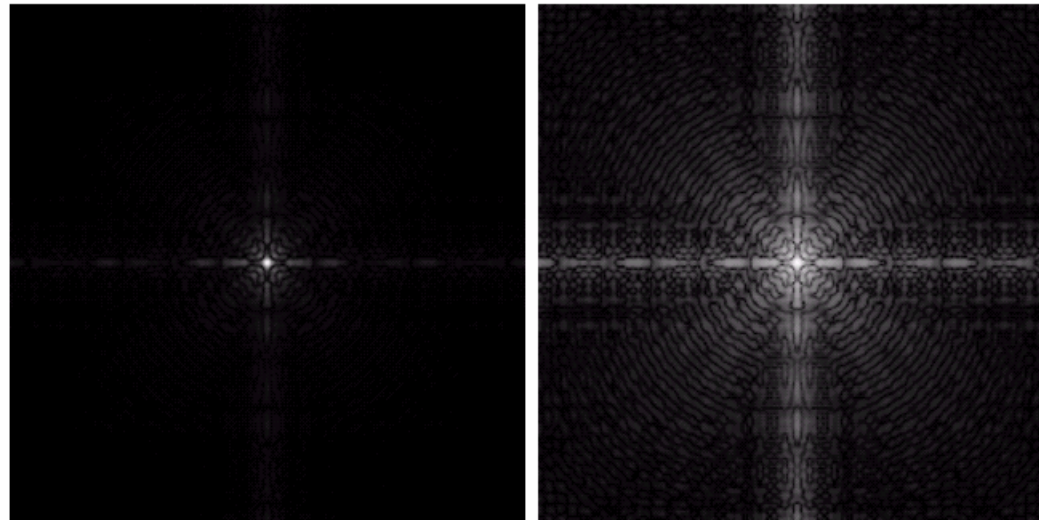


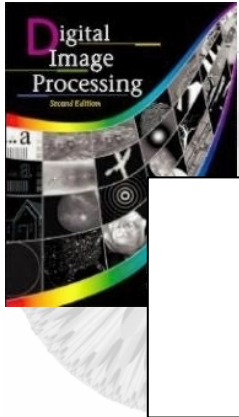
## Chapter 3

# Image Enhancement in the Spatial Domain

a b

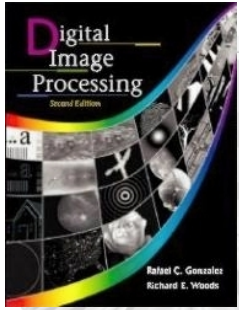
**FIGURE 3.5**  
(a) Fourier spectrum.  
(b) Result of applying the log transformation given in Eq. (3.2-2) with  $c = 1$ .





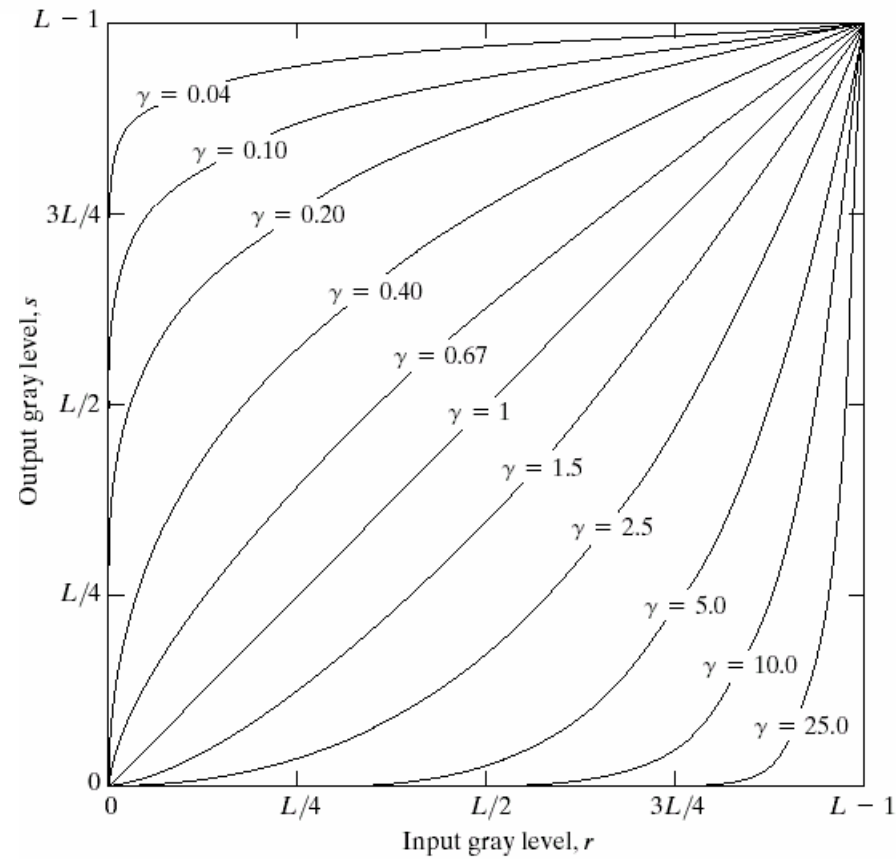
# Enhancement

- 3.  $s = c r^r$

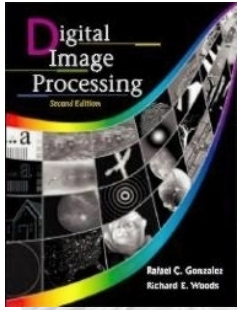


# Chapter 3

## Image Enhancement in the Spatial Domain



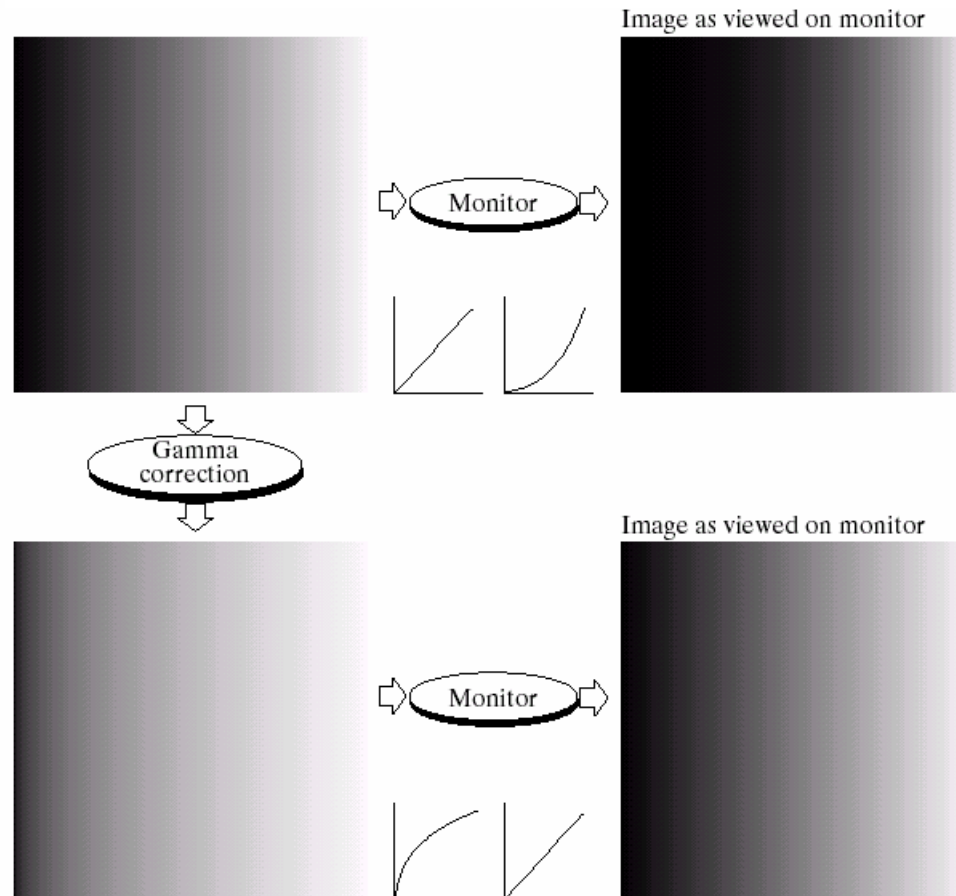
**FIGURE 3.6** Plots of the equation  $s = cr^\gamma$  for various values of  $\gamma$  ( $c = 1$  in all cases).

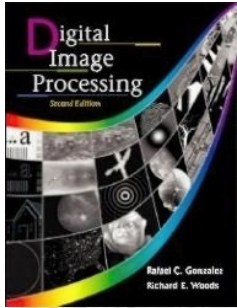


# Chapter 3 Image Enhancement in the Spatial Domain

a b  
c d

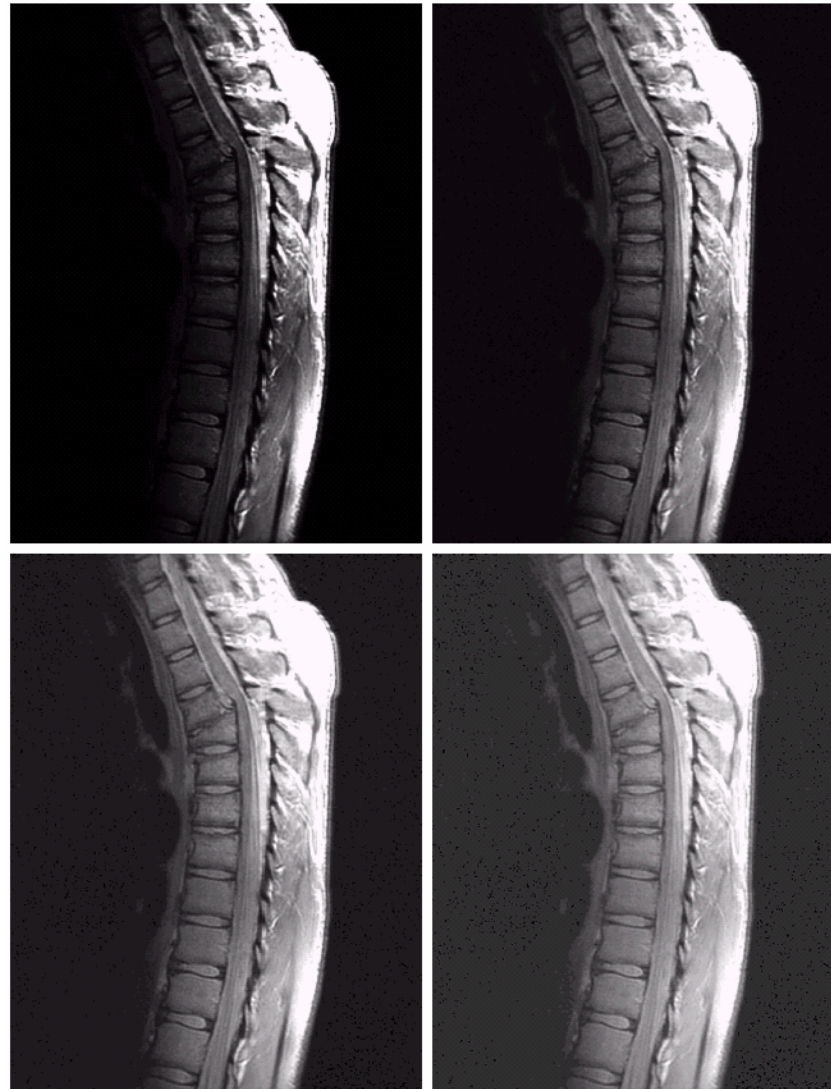
**FIGURE 3.7**  
(a) Linear-wedge gray-scale image.  
(b) Response of monitor to linear wedge.  
(c) Gamma-corrected wedge.  
(d) Output of monitor.





## Chapter 3

# Image Enhancement in the Spatial Domain

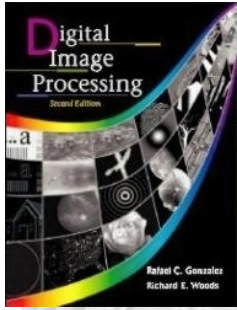


a b  
c d

**FIGURE 3.8**

(a) Magnetic resonance (MR) image of a fractured human spine. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with  $c = 1$  and  $\gamma = 0.6, 0.4,$  and  $0.3,$  respectively. (Original image for this example courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)



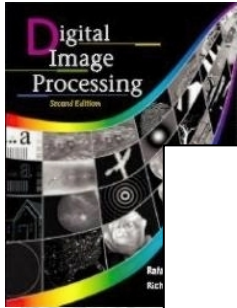


## Chapter 3 Image Enhancement in the Spatial Domain

a b  
c d

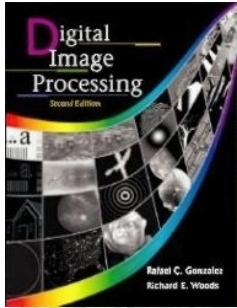
**FIGURE 3.9**  
(a) Aerial image.  
(b)–(d) Results of  
applying the  
transformation in  
Eq. (3.2-3) with  
 $c = 1$  and  
 $\gamma = 3.0, 4.0,$  and  
 $5.0,$  respectively.  
(Original image  
for this example  
courtesy of  
NASA.)





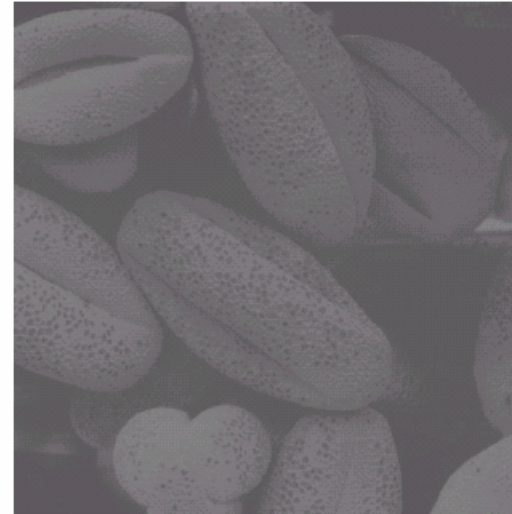
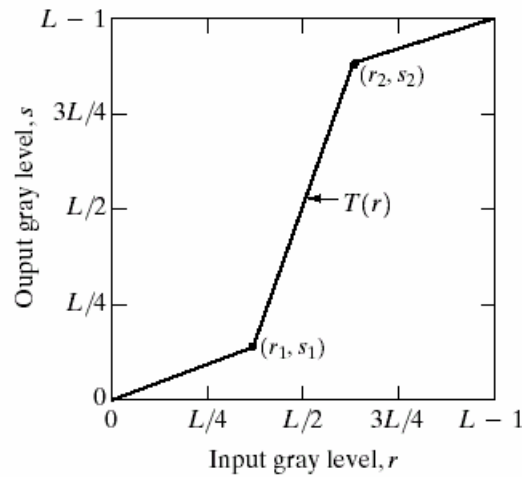
# Enhancement

- 4 . Piecewise Linear Transformation  
-- Contrast stretching



# Chapter 3

## Image Enhancement in the Spatial Domain

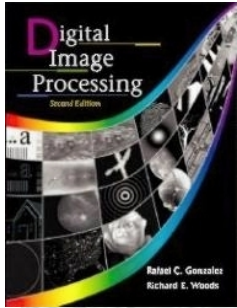


a b  
c d

**FIGURE 3.10** Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

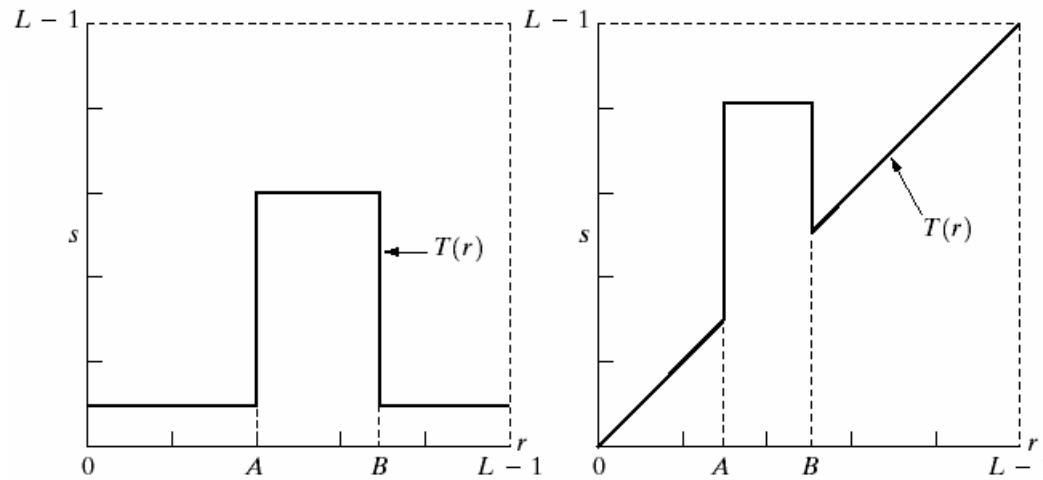






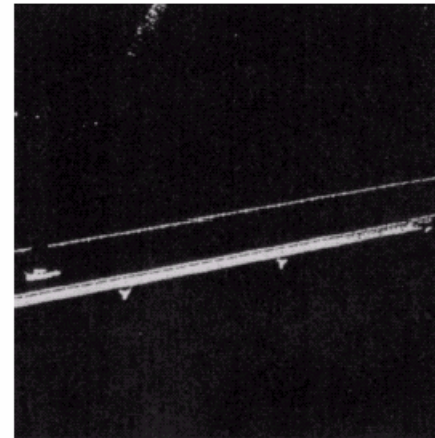
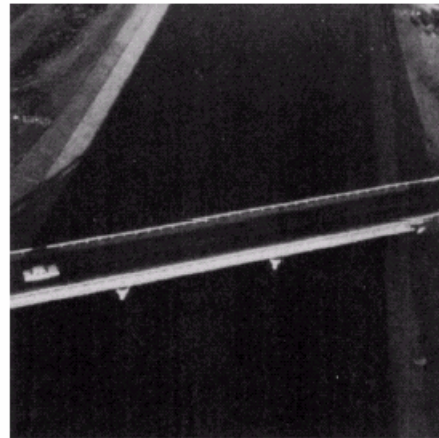
# Chapter 3

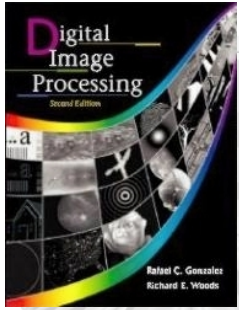
## Image Enhancement in the Spatial Domain



a	b
c	d

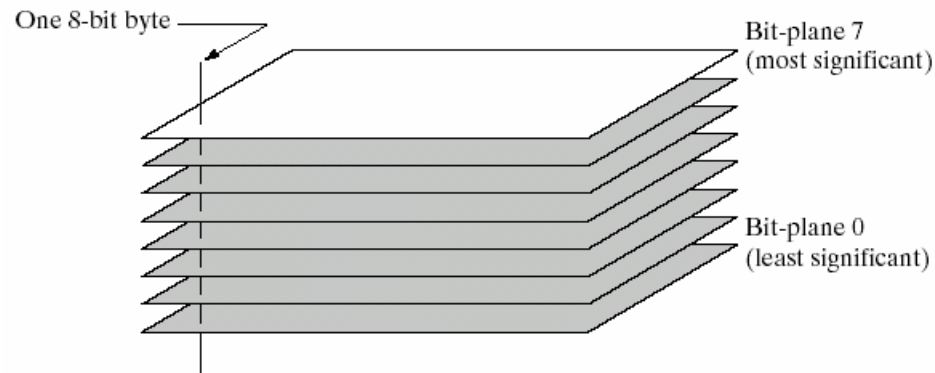
**FIGURE 3.11**  
 (a) This transformation highlights range  $[A, B]$  of gray levels and reduces all others to a constant level.  
 (b) This transformation highlights range  $[A, B]$  but preserves all other levels.  
 (c) An image.  
 (d) Result of using the transformation in (a).



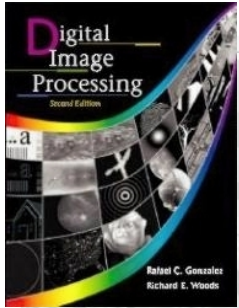


## Chapter 3

# Image Enhancement in the Spatial Domain

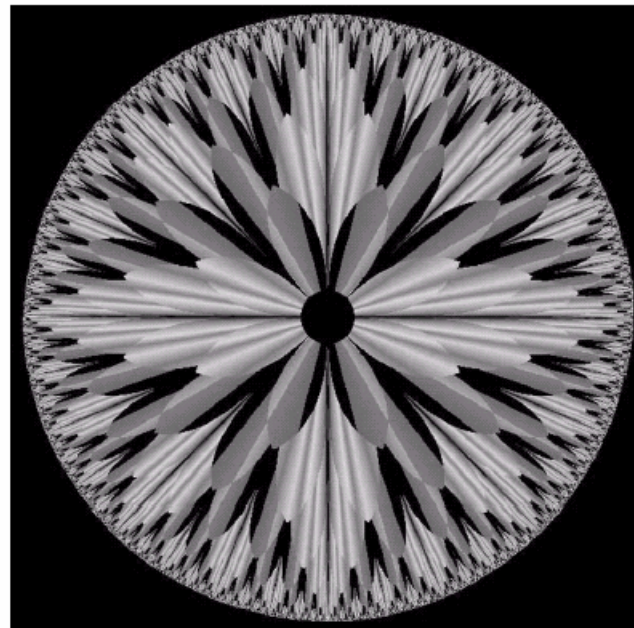


**FIGURE 3.12**  
Bit-plane  
representation of  
an 8-bit image.

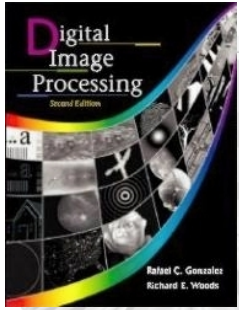


## Chapter 3

# Image Enhancement in the Spatial Domain

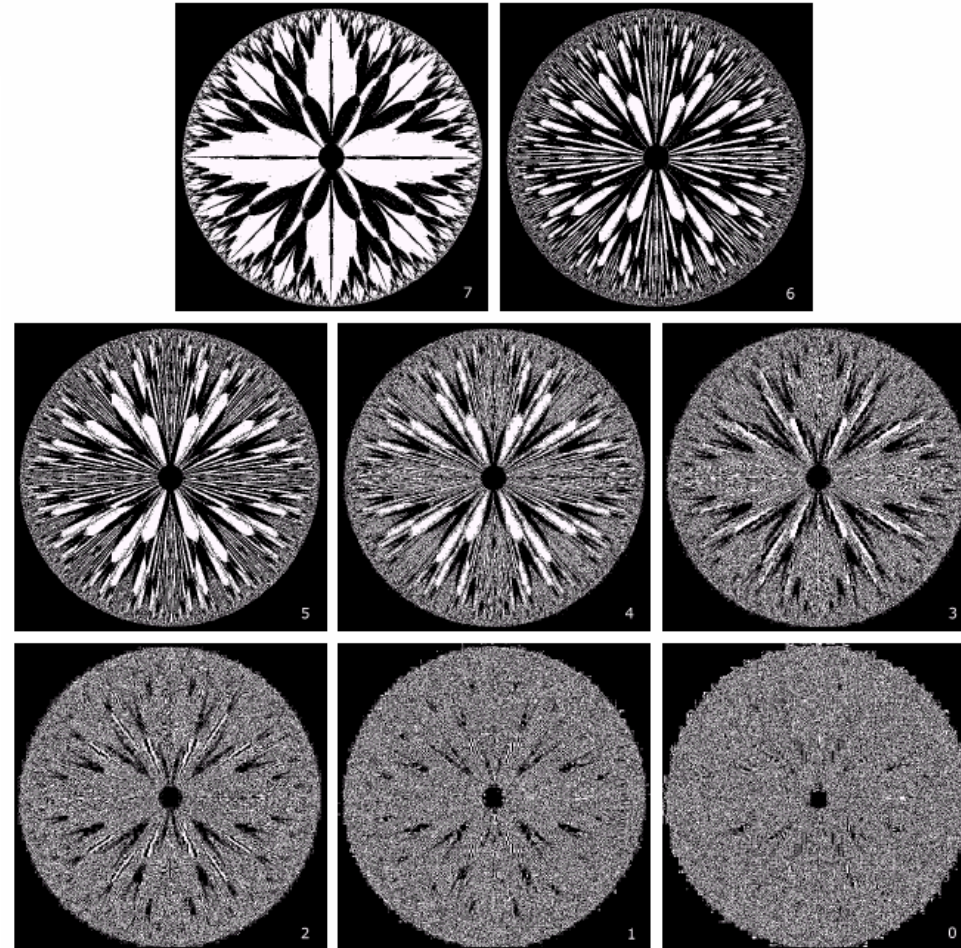


**FIGURE 3.13** An 8-bit fractal image. (A fractal is an image generated from mathematical expressions). (Courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA.)



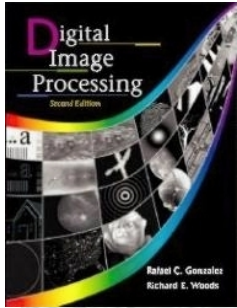
## Chapter 3

# Image Enhancement in the Spatial Domain



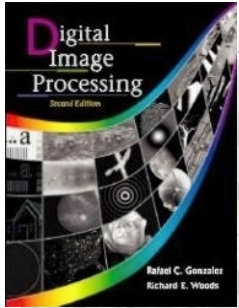
**FIGURE 3.14** The eight bit planes of the image in Fig. 3.13. The number at the bottom, right of each image identifies the bit plane.



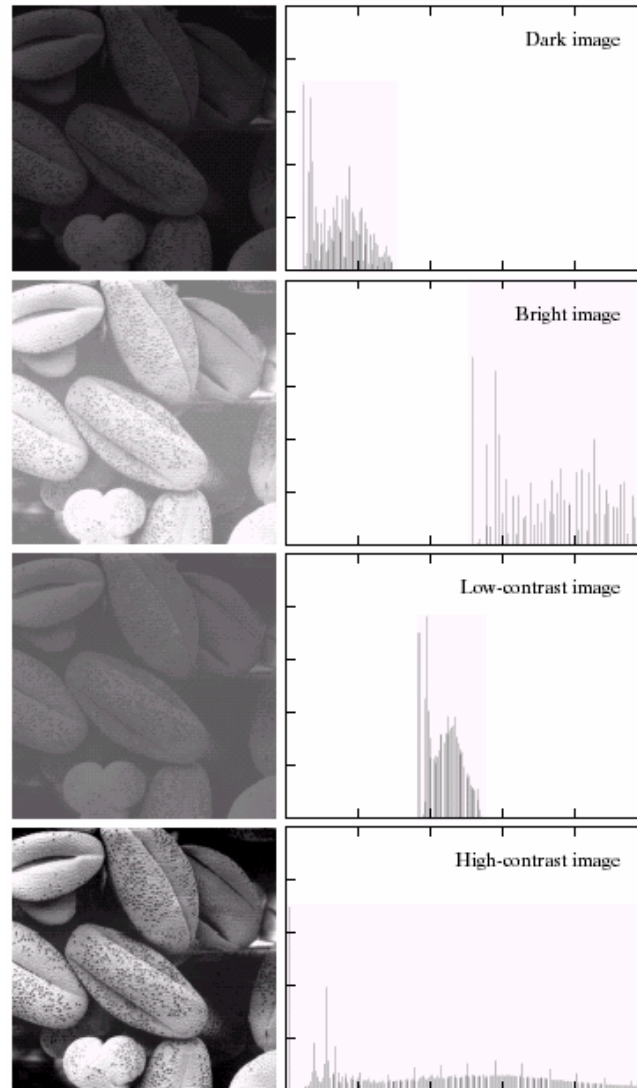


## 3.3 Histogram Processing

- Grey level:  $0 \sim L-1$
- $P(r_k) = n_k/n$
- $r_k$ : the  $k$ th grey level
- $n_k$ : the # of pixel with grey level  $k$
- $n$ : the # of total pixel
- Histogram:  $P(r_k) = n_k/n$

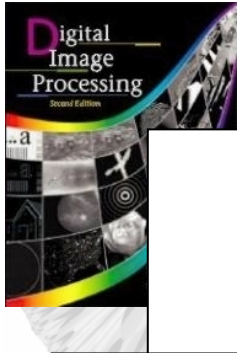


## Chapter 3 Image Enhancement in the Spatial Domain

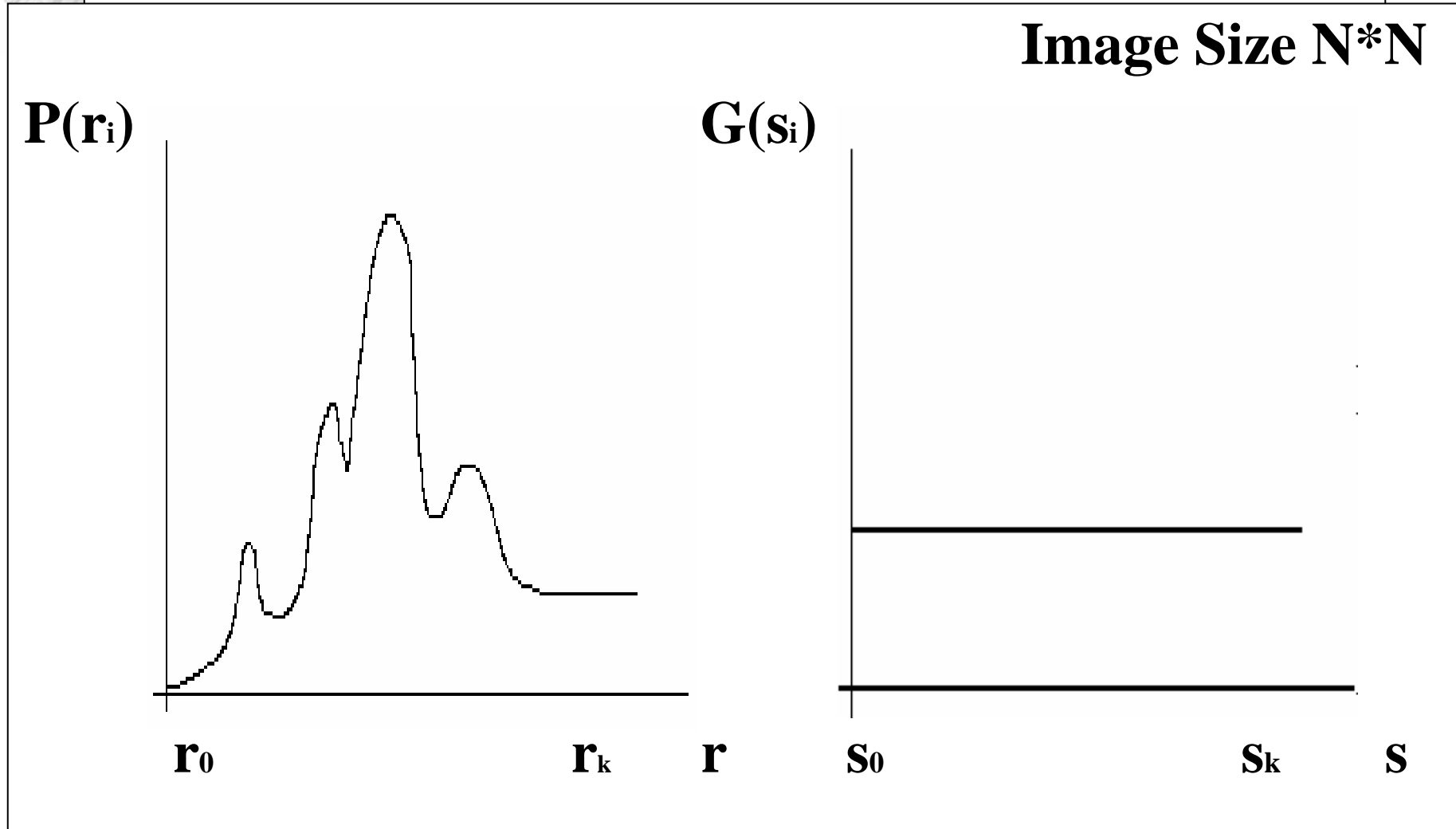


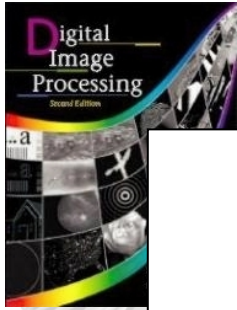
a b

**FIGURE 3.15** Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)



# Histogram Equalization(1)





# Histogram Equalization (2)

$$f(r) = S$$

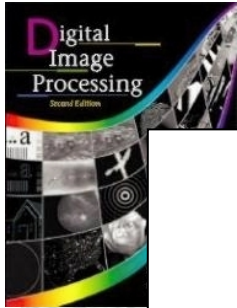
Since

$$\sum_{i=0}^k P(r_i) = \sum_{i=0}^k G(s_i)$$

We know that  $G$  is equalized if

$$\forall i, G(s_i) = \frac{N^2}{S_K - S_0}$$



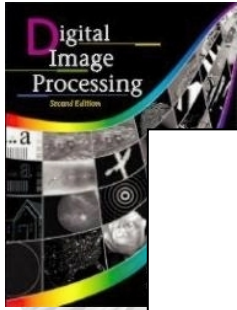


# Histogram Equalization (3)

**From probability CDF**

$$\int_{S_0}^S \frac{N^2}{S_K - S_0} ds = \int_{r_0}^r P(s) ds$$

$$\frac{N^2(S - S_0)}{S_K - S_0} = \int_{r_0}^r P(s) ds$$

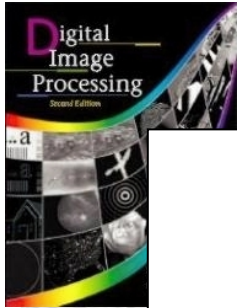


# Histogram Equalization (4)

$$S = \frac{S_K - S_0}{N^2} \int_{r_0}^r P(s) ds + S_0$$

**From Discrete format**

$$S = \frac{S_K - S_0}{N^2} \sum_{\bar{i} = r_0}^r P(\bar{i}) + S_0$$

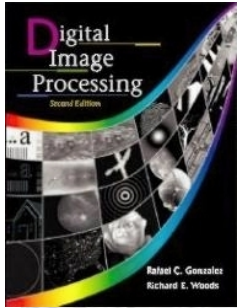


# Histogram Equalization (5)

$$S = f(r) = \frac{S_K - S_0}{N^2} \sum_{\bar{i} = r_0}^r P(\bar{i}) + S_0$$

EX:

$$S = f(100) = \frac{S_K - S_0}{N^2} \sum_{\bar{i} = r_0}^{100} P(\bar{i}) + S_0$$



- **Histogram Equalization**

$$S=T(r)$$

(a)  $T(r)$  = monotonic increasing

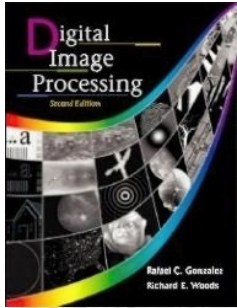
(b)  $0 \leq T(r) \leq 1$

$$r=T^{-1}(s)$$

$$P_d f = P_r(r) \cdot P_s(r)$$

$$P_s(s) = [P_r(r) dr/ds]_{r=T^{-1}(s)}$$

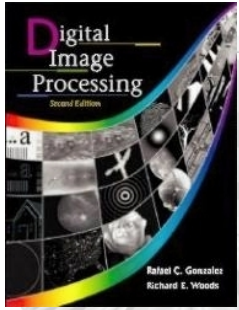
**Controlling the pdf for transfer function**



$$\text{CDF } S=T(r)= \int_0^r \text{Pr}(w)dw \quad 0 \leq r \leq 1$$

**Note:**

**Using a transfer function equals to the CDF of r produce an image whose grey level has a uniform density and increase the dynamic range of the pixels**



## • Histogram Equalization

- Global

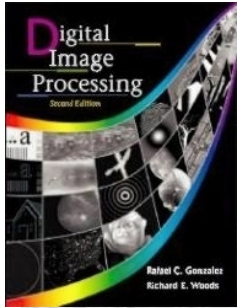
- Local

**Pdf:  $P_r(r)$**

**Ex.**

**Assume the pdf  $P_r(r)$  of a image is**

**$P_r(r) = -2r + 2, 0 \leq r \leq 1$**



$$S = T(r) = \int_0^r \text{Pr}(w)dw$$

$$= \int_0^r (-2w + 2)dw$$

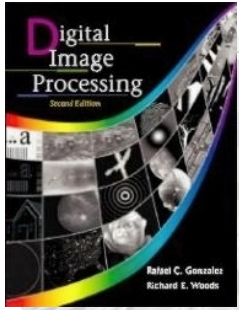
$$= -w^2 + 2w \Big|_0^r$$

$$= -r^2 + 2r$$

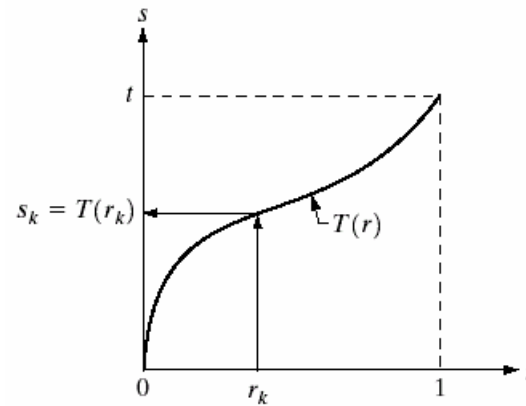
$$\therefore S = -r^2 + 2r$$

$$r = T^{-1}(s) = 1 \pm \sqrt{1-s}$$

$$\therefore r = 1 - \sqrt{1-s}$$

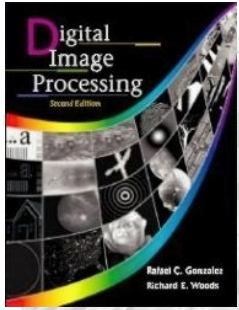


## Chapter 3 Image Enhancement in the Spatial Domain



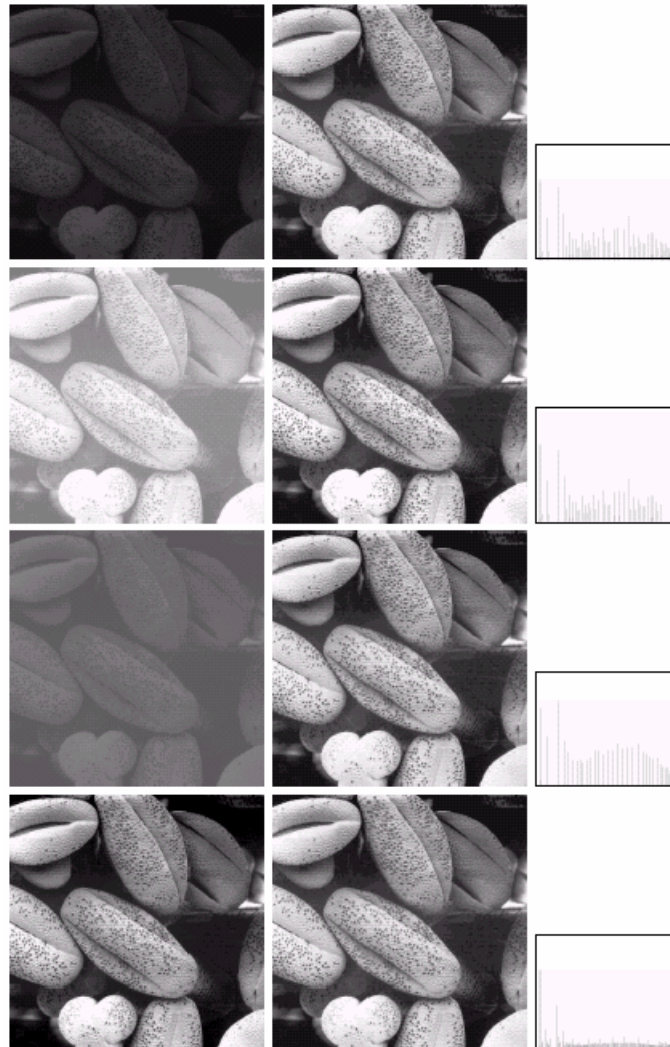
**FIGURE 3.16** A gray-level transformation function that is both single valued and monotonically increasing.





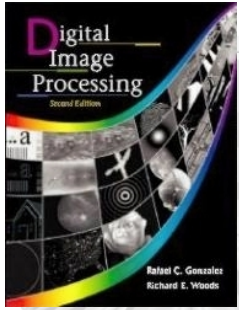
# Chapter 3

## Image Enhancement in the Spatial Domain



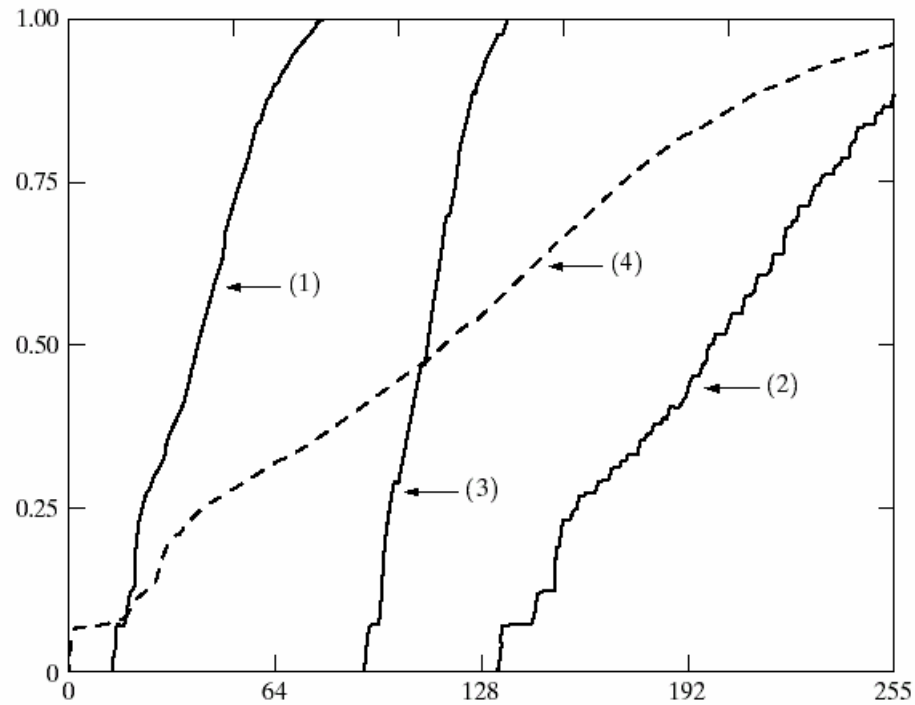
a b c

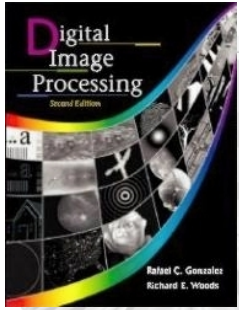
FIGURE 3.17 (a) Images from Fig. 3.15. (b) Results of histogram equalization. (c) Corresponding histograms.



## Chapter 3 Image Enhancement in the Spatial Domain

**FIGURE 3.18**  
Transformation functions (1) through (4) were obtained from the histograms of the images in Fig.3.17(a), using Eq. (3.3-8).





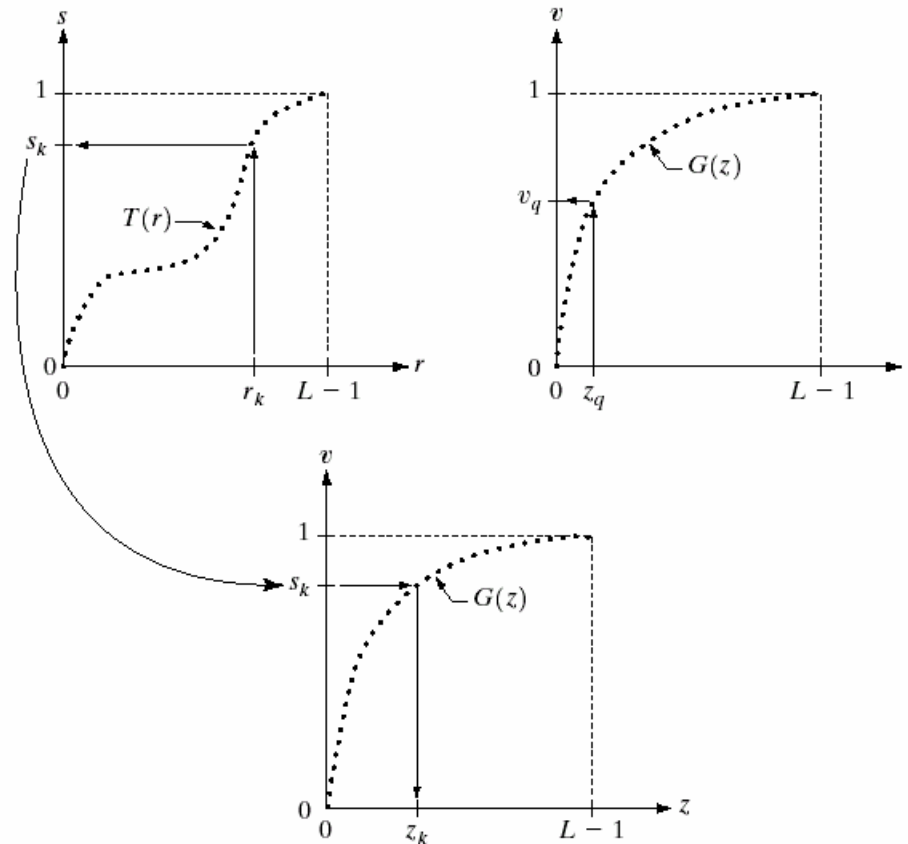
# Chapter 3

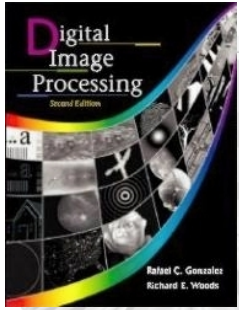
## Image Enhancement in the Spatial Domain

a b  
c

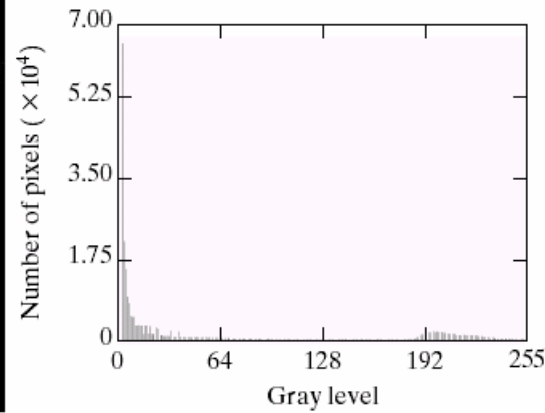
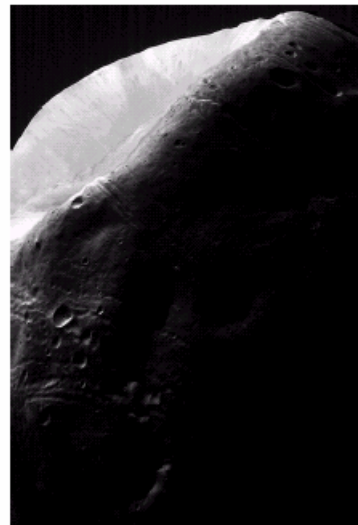
**FIGURE 3.19**

(a) Graphical interpretation of mapping from  $r_k$  to  $s_k$  via  $T(r)$ .  
(b) Mapping of  $z_q$  to its corresponding value  $v_q$  via  $G(z)$ .  
(c) Inverse mapping from  $s_k$  to its corresponding value of  $z_k$ .



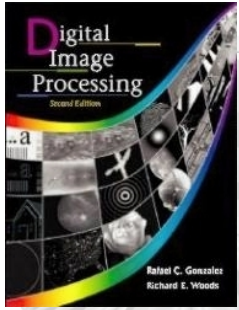


## Chapter 3 Image Enhancement in the Spatial Domain



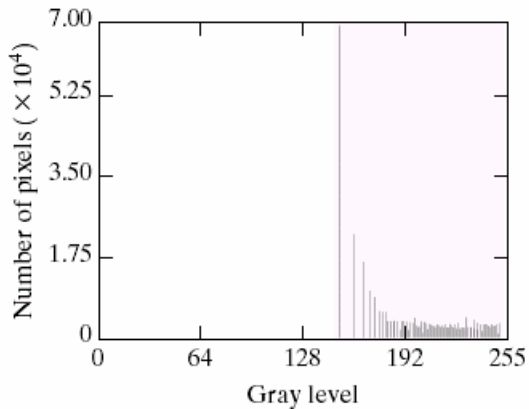
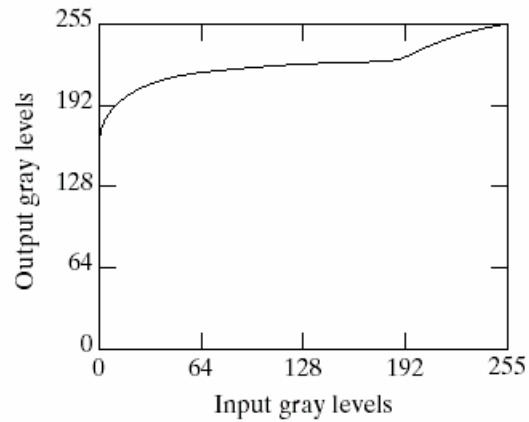
a b

**FIGURE 3.20** (a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor*. (b) Histogram. (Original image courtesy of NASA.)



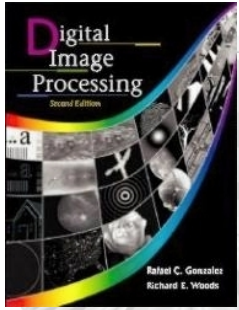
# Chapter 3

## Image Enhancement in the Spatial Domain



a b  
c

**FIGURE 3.21**  
(a) Transformation function for histogram equalization.  
(b) Histogram-equalized image (note the washed-out appearance).  
(c) Histogram of (b).

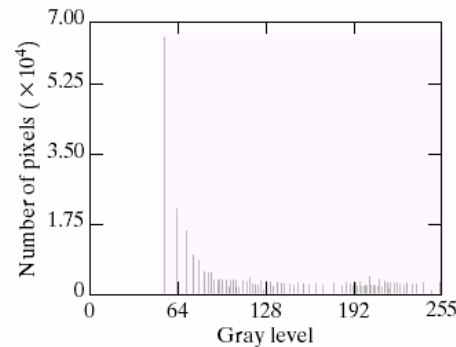
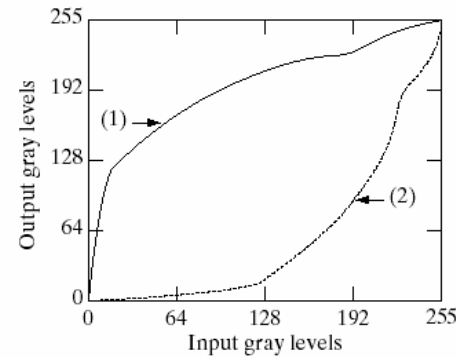
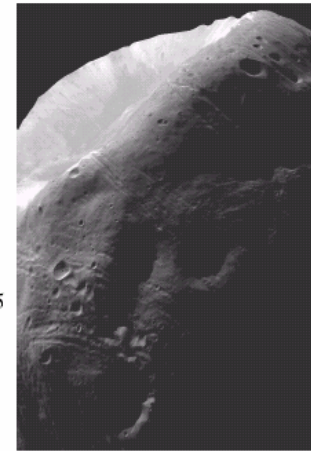
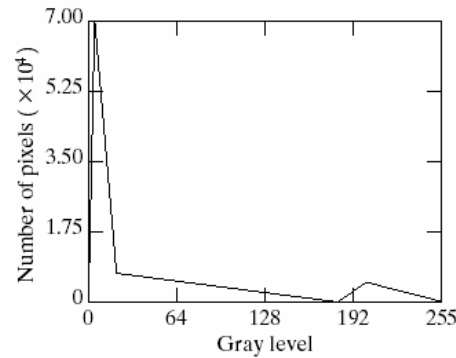


# Chapter 3

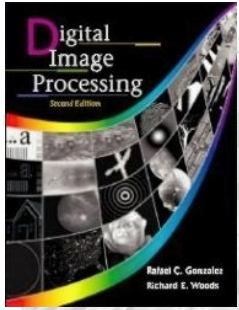
## Image Enhancement in the Spatial Domain

a c  
b  
d

**FIGURE 3.22**  
(a) Specified histogram.  
(b) Curve (1) is from Eq. (3.3-14), using the histogram in (a); curve (2) was obtained using the iterative procedure in Eq. (3.3-17).  
(c) Enhanced image using mappings from curve (2).  
(d) Histogram of (c).

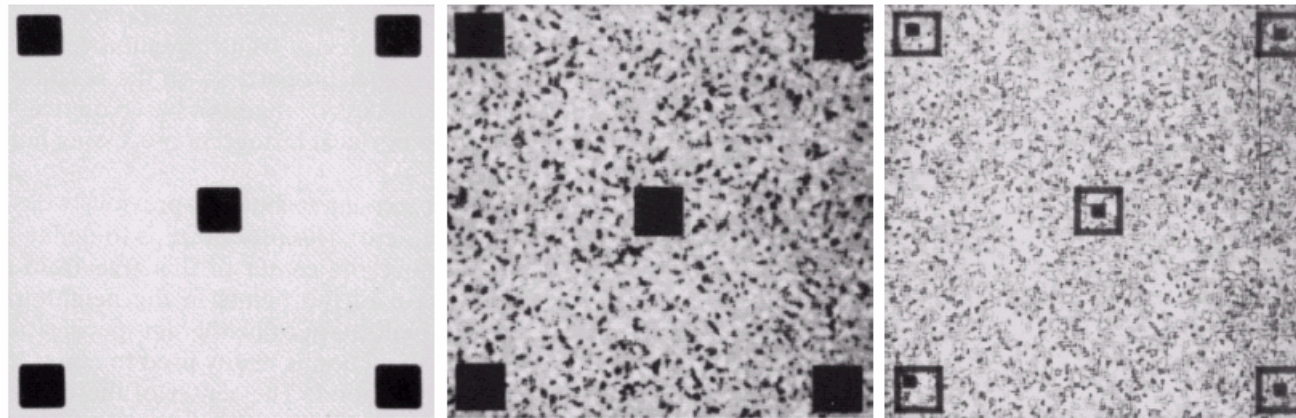






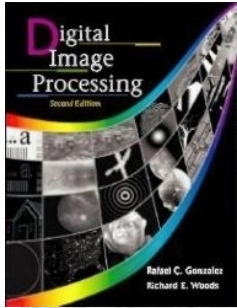
## Chapter 3

# Image Enhancement in the Spatial Domain



a b c

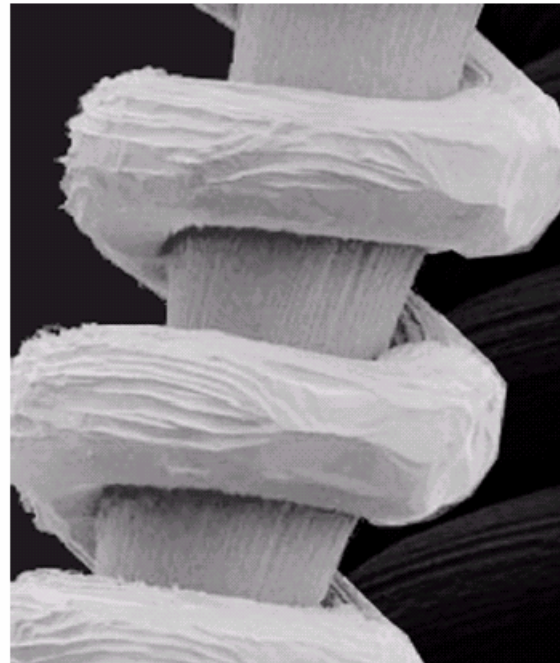
**FIGURE 3.23** (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a  $7 \times 7$  neighborhood about each pixel.



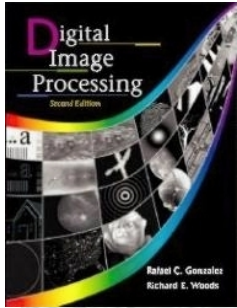
## Chapter 3

# Image Enhancement in the Spatial Domain

**FIGURE 3.24** SEM image of a tungsten filament and support, magnified approximately 130 $\times$ . (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene).

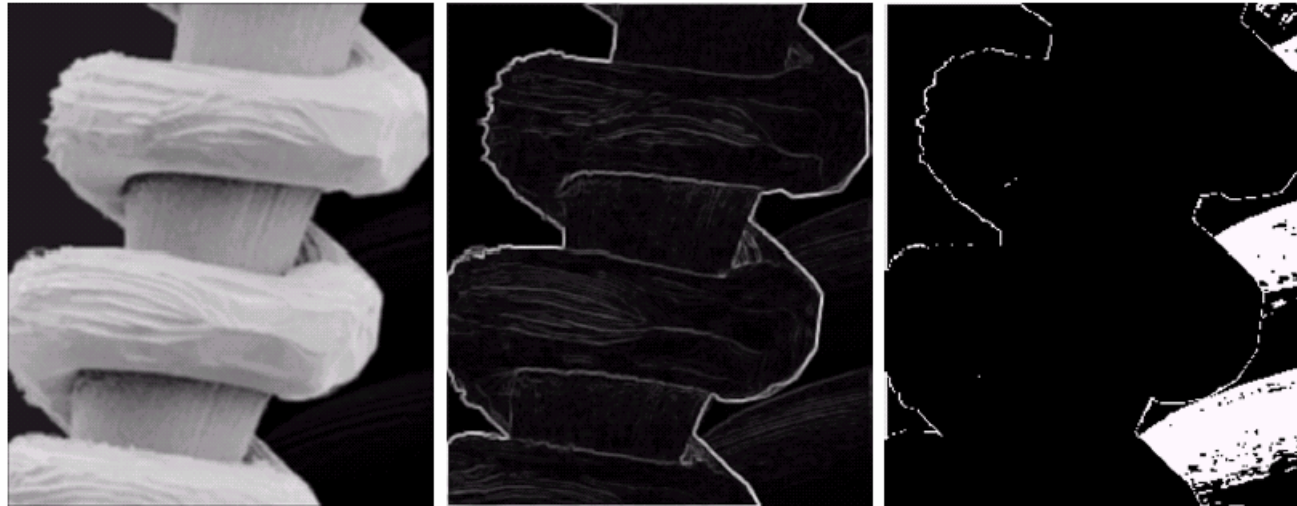






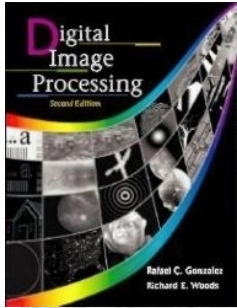
## Chapter 3

# Image Enhancement in the Spatial Domain



a b c

**FIGURE 3.25** (a) Image formed from all local means obtained from Fig. 3.24 using Eq. (3.3-21). (b) Image formed from all local standard deviations obtained from Fig. 3.24 using Eq. (3.3-22). (c) Image formed from all multiplication constants used to produce the enhanced image shown in Fig. 3.26.

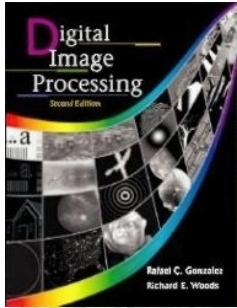


## Chapter 3

# Image Enhancement in the Spatial Domain



**FIGURE 3.26**  
Enhanced SEM image. Compare with Fig. 3.24. Note in particular the enhanced area on the right side of the image.



*(1) Substraction*

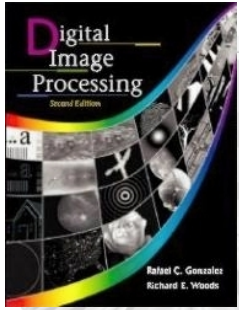
$$g(x, y) = f(x, y) - h(x, y)$$

*(2) Averaging*

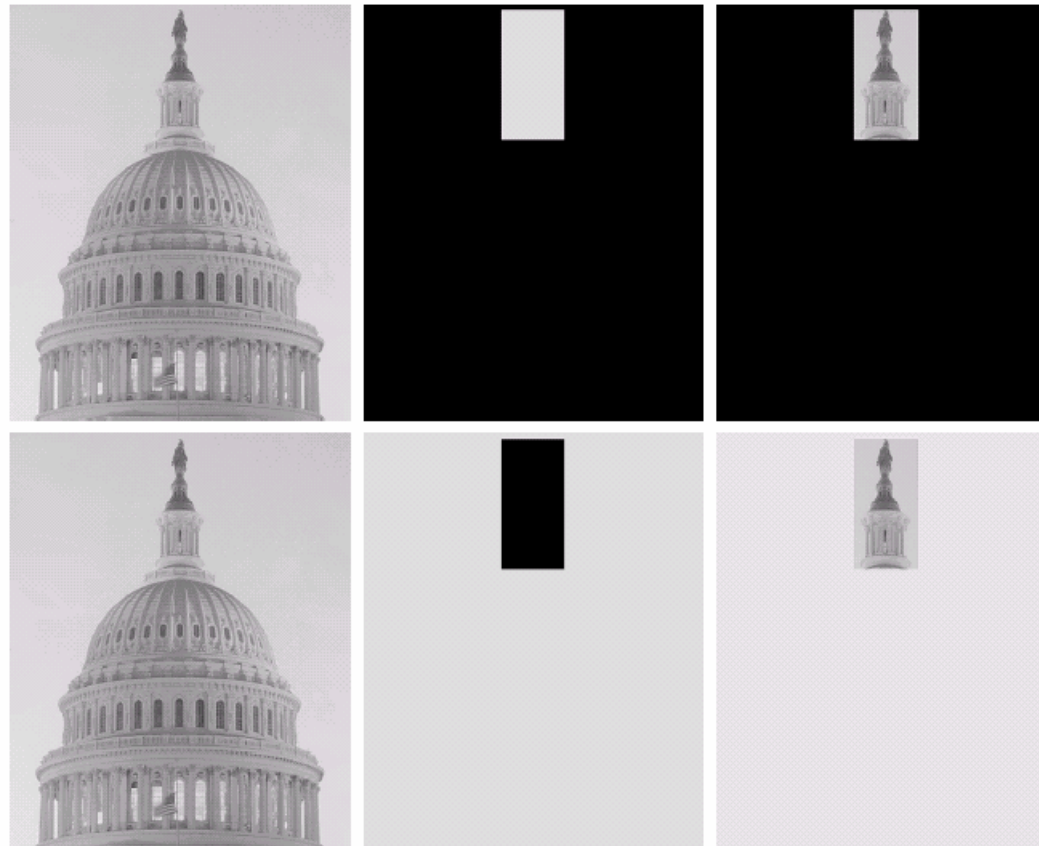
$$g(x, y) = f(x, y) + \eta(x, y)$$

$\eta(x, y)$     *noise*

$$\bar{g}(x, y) = \frac{1}{k} \sum_{i=1}^k g_i(x, y)$$



## Chapter 3 Image Enhancement in the Spatial Domain

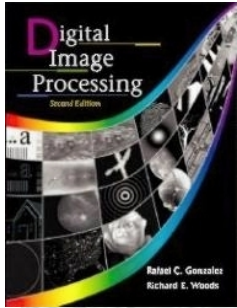


a	b	c
d	e	f

**FIGURE 3.27**

(a) Original image. (b) AND image mask. (c) Result of the AND operation on images (a) and (b). (d) Original image. (e) OR image mask. (f) Result of operation OR on images (d) and (e).



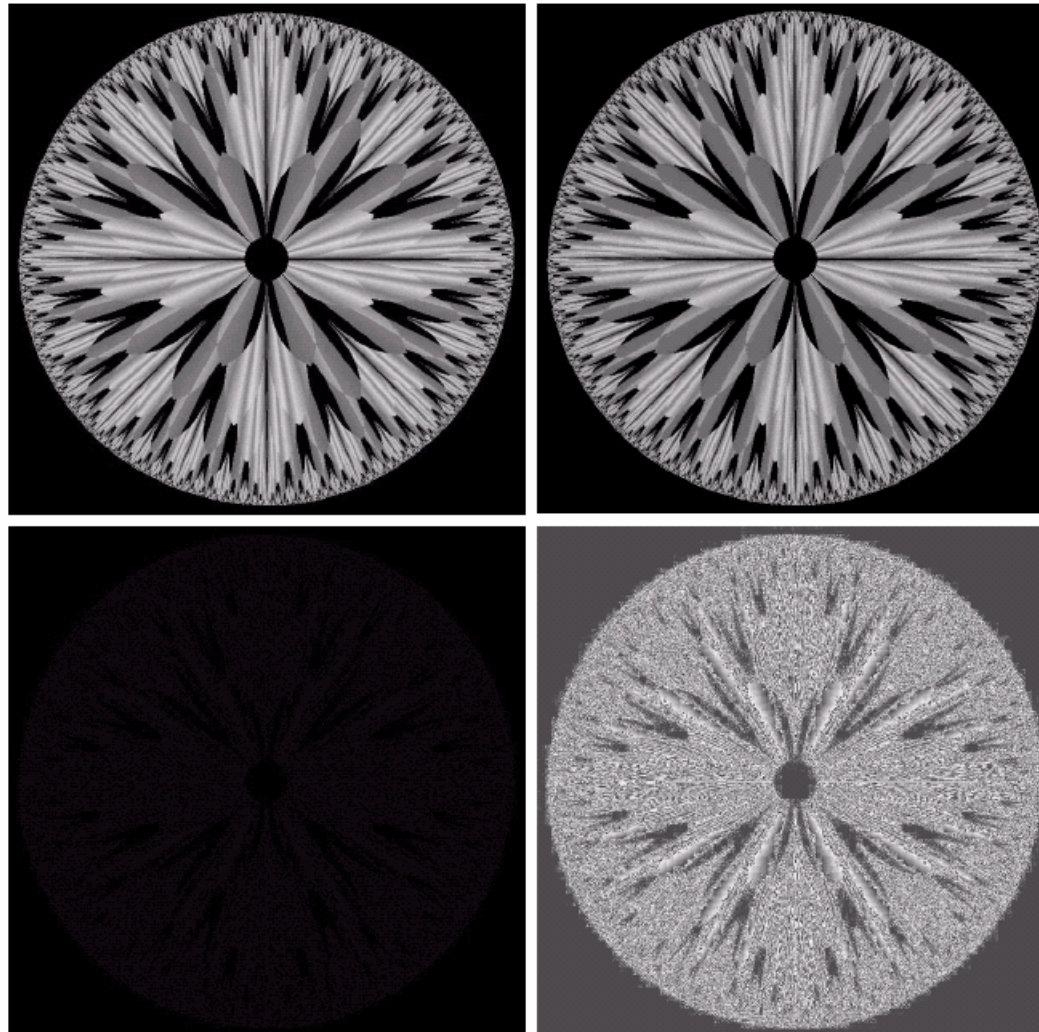


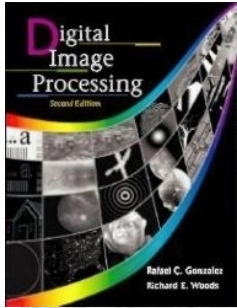
## Chapter 3

# Image Enhancement in the Spatial Domain

a b  
c d

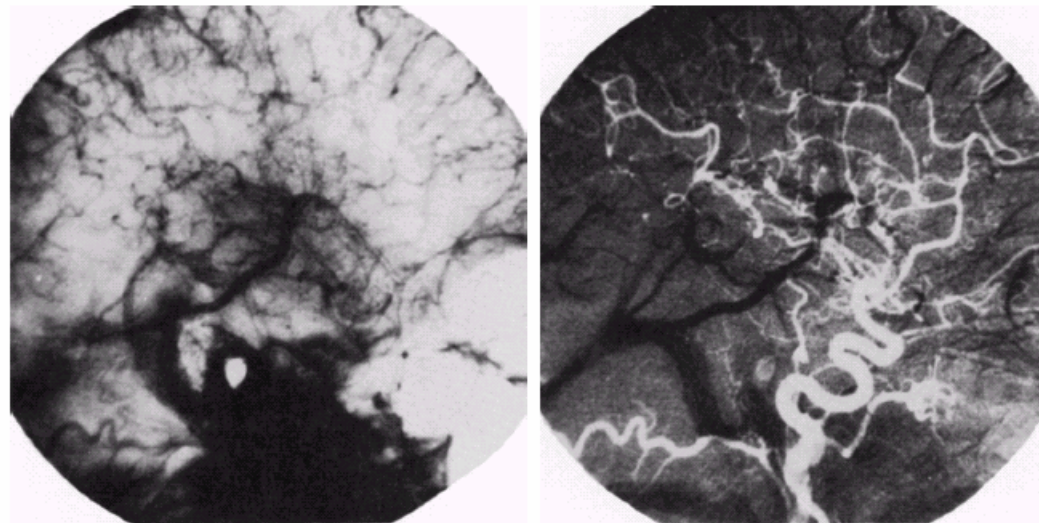
**FIGURE 3.28**  
(a) Original fractal image.  
(b) Result of setting the four lower-order bit planes to zero.  
(c) Difference between (a) and (b).  
(d) Histogram-equalized difference image. (Original image courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA).





## Chapter 3

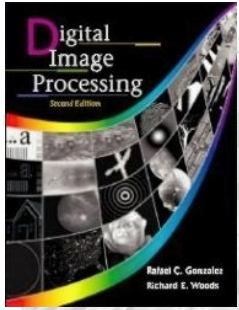
# Image Enhancement in the Spatial Domain



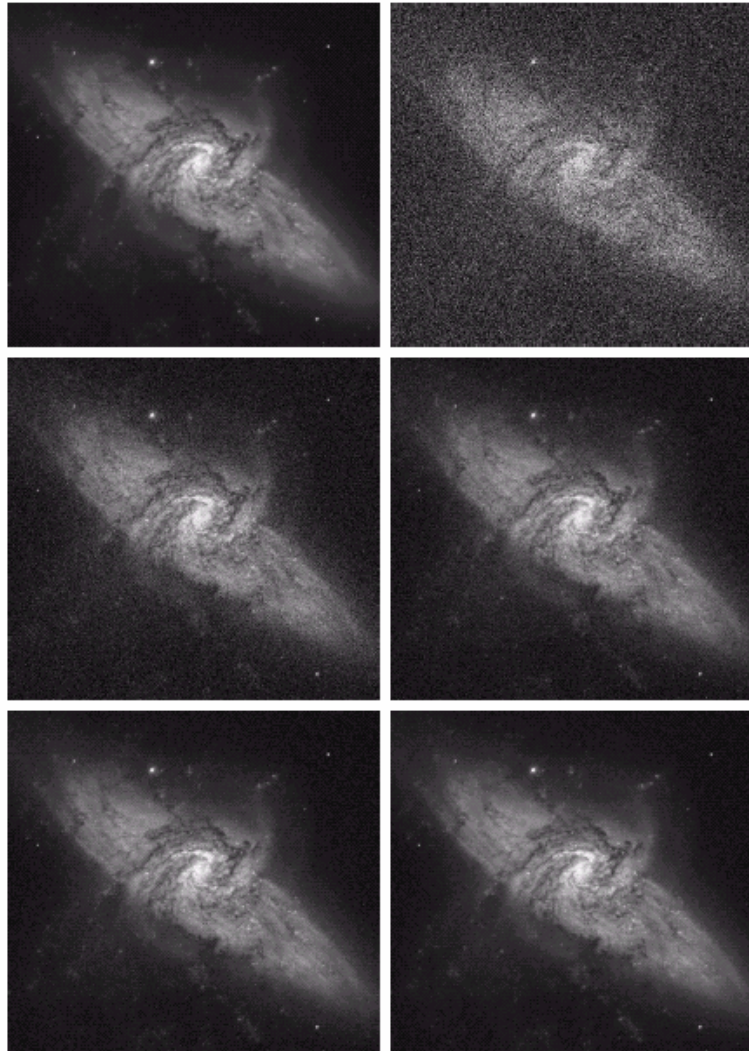
a b

**FIGURE 3.29**  
Enhancement by image subtraction.  
(a) Mask image.  
(b) An image (taken after injection of a contrast medium into the bloodstream) with mask subtracted out.





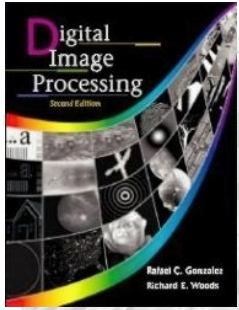
## Chapter 3 Image Enhancement in the Spatial Domain



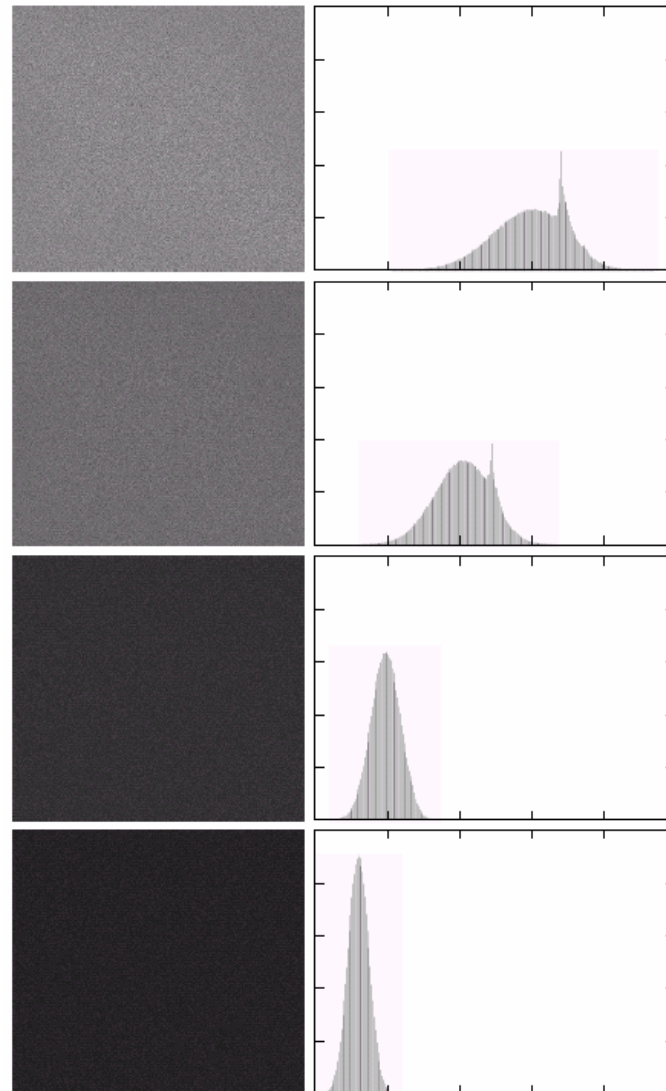
a b  
c d  
e f

**FIGURE 3.30** (a) Image of Galaxy Pair NGC 3314. (b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels. (c)–(f) Results of averaging  $K = 8, 16, 64,$  and  $128$  noisy images. (Original image courtesy of NASA.)

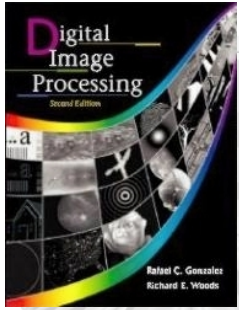




## Chapter 3 Image Enhancement in the Spatial Domain



a b  
**FIGURE 3.31**  
(a) From top to bottom:  
Difference images  
between  
Fig. 3.30(a) and  
the four images in  
Figs. 3.30(c)  
through (f),  
respectively.  
(b) Corresponding  
histograms.



$$g(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(x, y) f(x+s, y+t)$$

(1) Smooth Filtering (Low pass filter)

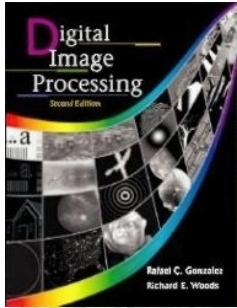
- noise reduction, blurring

(2) Sharpening Filtering (high pass filter)

- highlight details

eg.

$$\begin{array}{ccc} -\frac{1}{9} & -\frac{1}{9} & -\frac{1}{9} \\ -\frac{1}{9} & \frac{8}{9} & -\frac{1}{9} \\ -\frac{1}{9} & -\frac{1}{9} & -\frac{1}{9} \end{array}$$



### (3) Derivative Filter

- edge enhancement

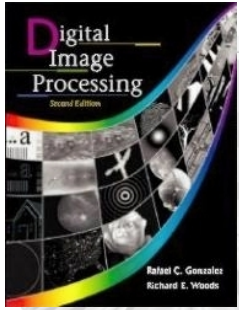
eg. Laplacian

### (4) Highboost Filtering

$$\text{Highboost} = A \bullet \text{orig} - \text{Lowpass} = (A-1) \bullet \text{orig} + \text{Highpass}$$

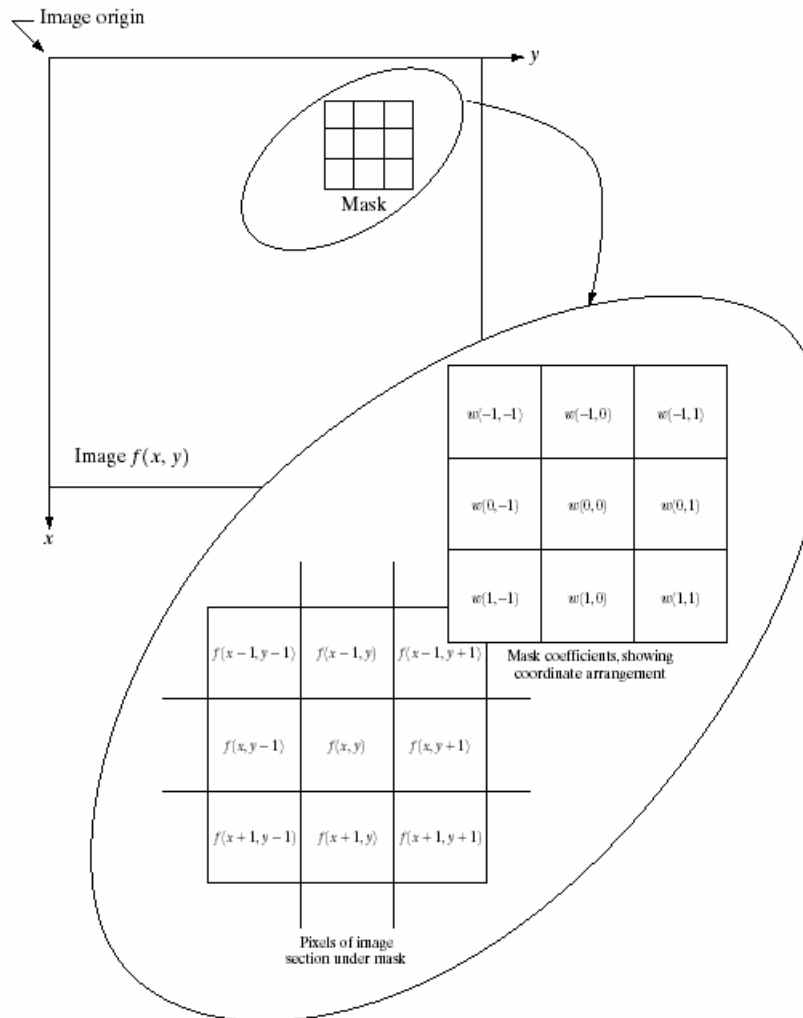
when  $A = 1$ , highpass

$A > 1$ , part of the original image is added back to highpass

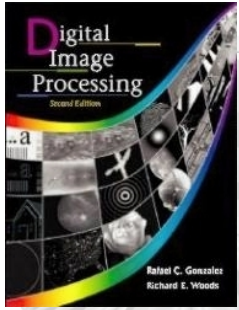


# Chapter 3

## Image Enhancement in the Spatial Domain



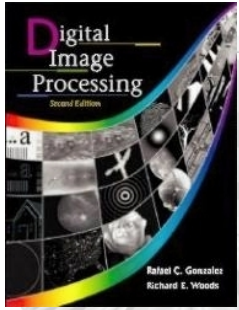
**FIGURE 3.32** The mechanics of spatial filtering. The magnified drawing shows a  $3 \times 3$  mask and the image section directly under it; the image section is shown displaced out from under the mask for ease of readability.



## Chapter 3 Image Enhancement in the Spatial Domain

**FIGURE 3.33**  
Another  
representation of  
a general  $3 \times 3$   
spatial filter mask.

$w_1$	$w_2$	$w_3$
$w_4$	$w_5$	$w_6$
$w_7$	$w_8$	$w_9$



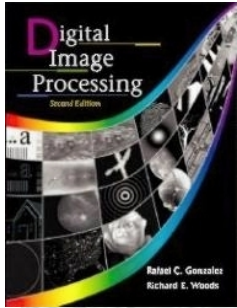
## Chapter 3 Image Enhancement in the Spatial Domain

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \quad \frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

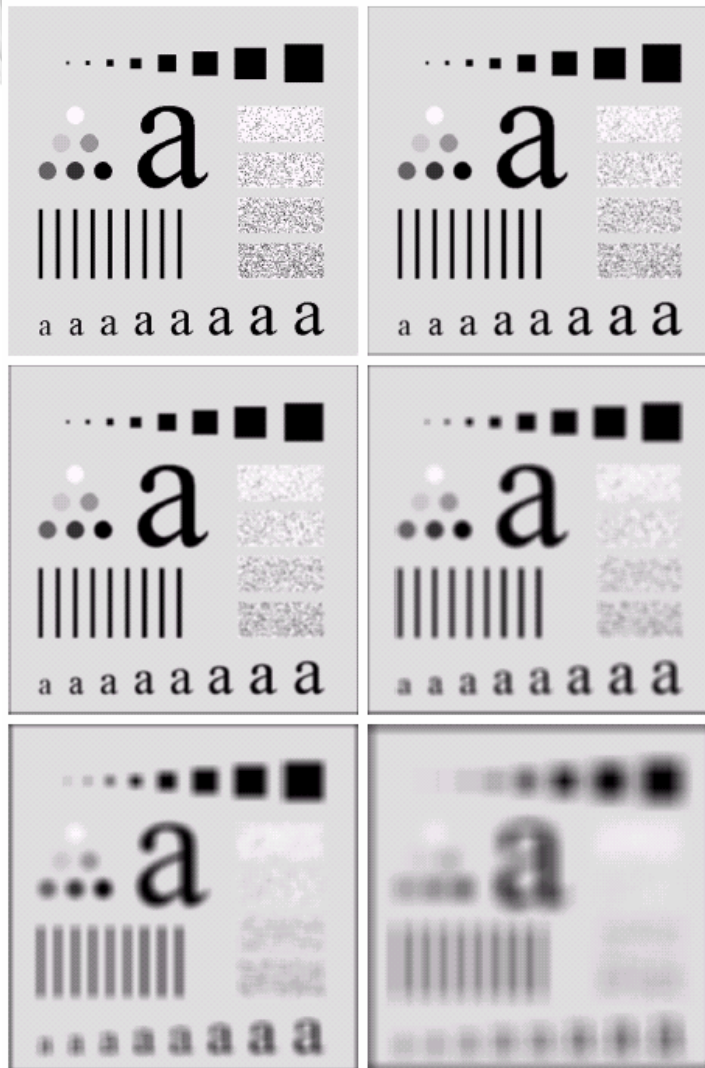
a b

**FIGURE 3.34** Two  $3 \times 3$  smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to the sum of the values of its coefficients, as is required to compute an average.

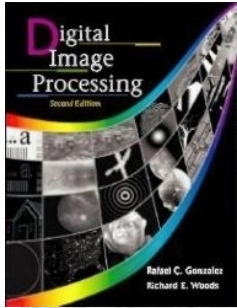




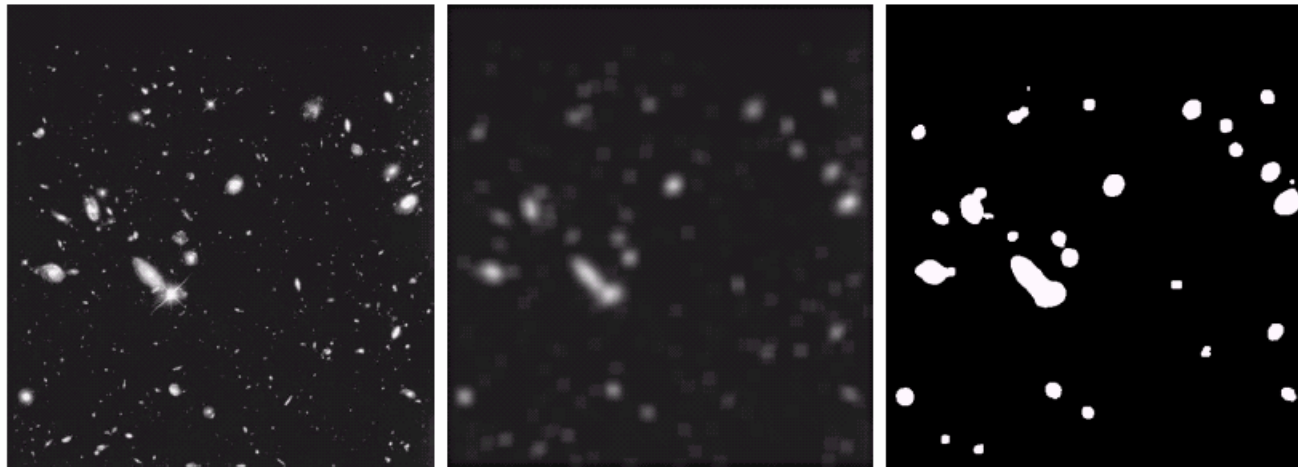
## Chapter 3 Image Enhancement in the Spatial Domain



**FIGURE 3.35** (a) Original image, of size  $500 \times 500$  pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes  $n = 3, 5, 9, 15,$  and  $35$ , respectively. The black squares at the top are of sizes  $3, 5, 9, 15, 25, 35, 45,$  and  $55$  pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size  $50 \times 120$  pixels.

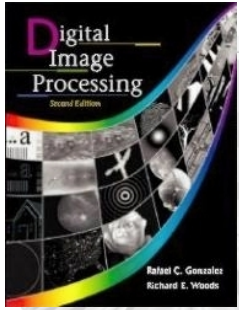


## Chapter 3 Image Enhancement in the Spatial Domain

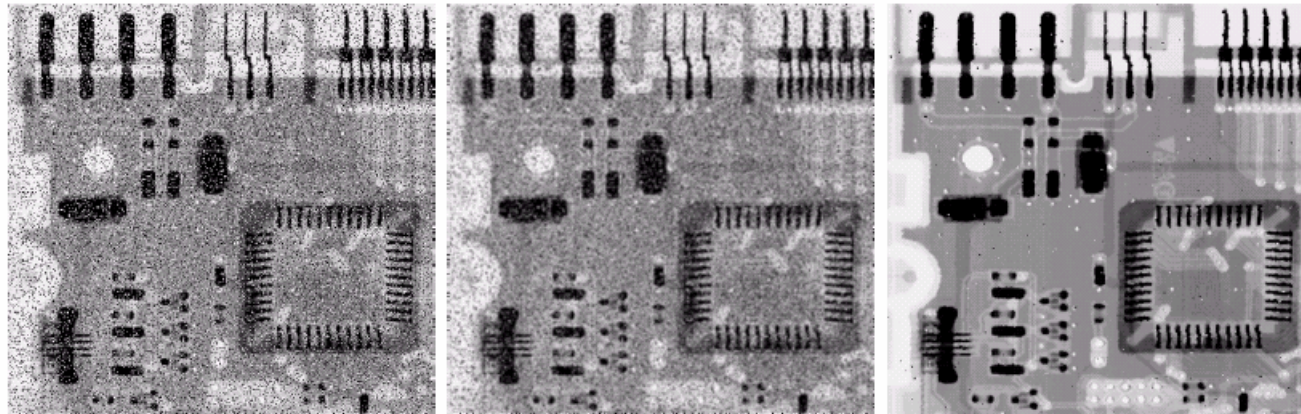


a b c

**FIGURE 3.36** (a) Image from the Hubble Space Telescope. (b) Image processed by a  $15 \times 15$  averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

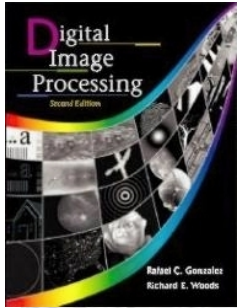


## Chapter 3 Image Enhancement in the Spatial Domain



a b c

**FIGURE 3.37** (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a  $3 \times 3$  averaging mask. (c) Noise reduction with a  $3 \times 3$  median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



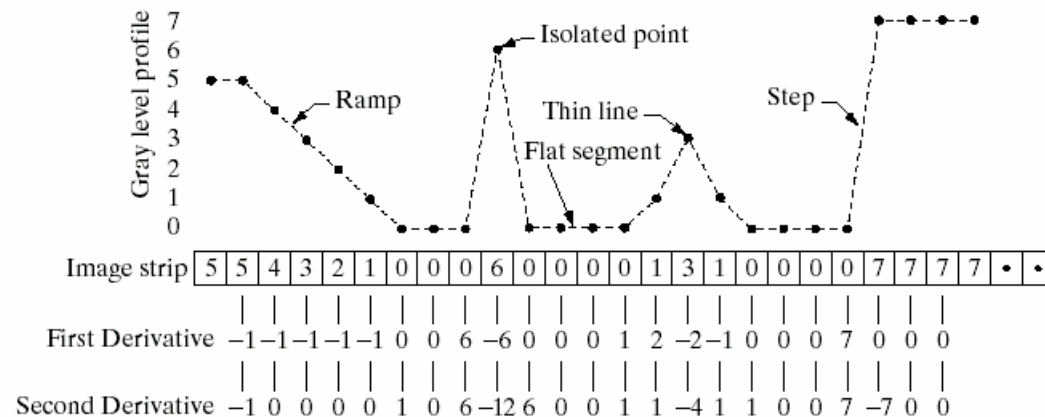
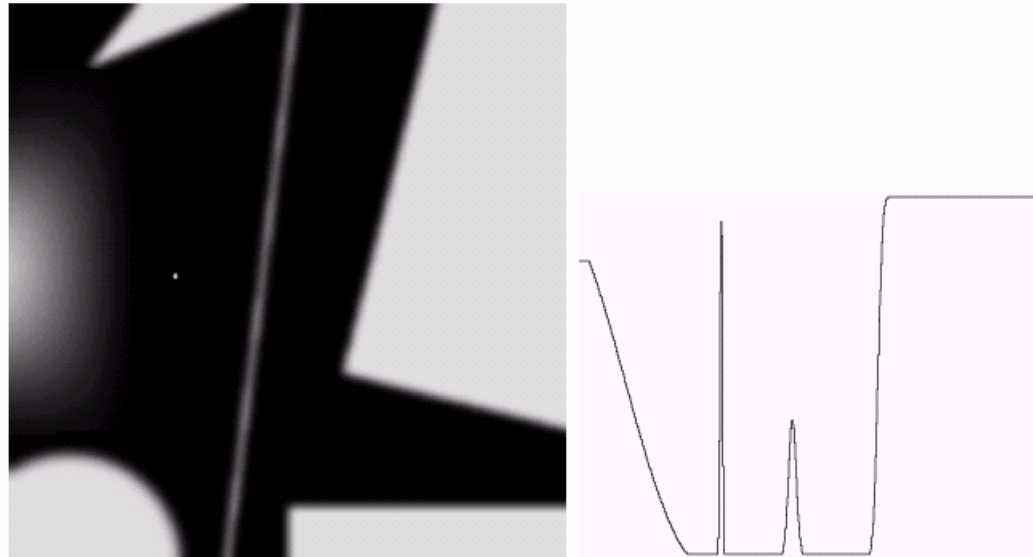
# Chapter 3

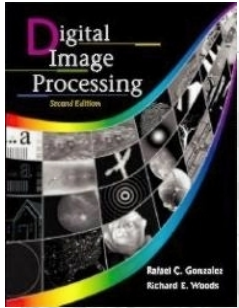
## Image Enhancement in the Spatial Domain

a b  
c

**FIGURE 3.38**

(a) A simple image. (b) 1-D horizontal gray-level profile along the center of the image and including the isolated noise point. (c) Simplified profile (the points are joined by dashed lines to simplify interpretation).





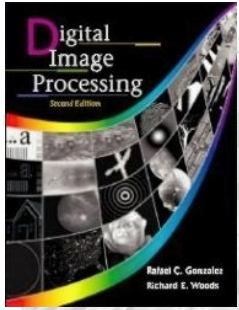
## Chapter 3 Image Enhancement in the Spatial Domain

0	1	0	1	1	1
1	-4	1	1	-8	1
0	1	0	1	1	1
0	-1	0	-1	-1	-1
-1	4	-1	-1	8	-1
0	-1	0	-1	-1	-1

a b  
c d

**FIGURE 3.39**  
(a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4).  
(b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.



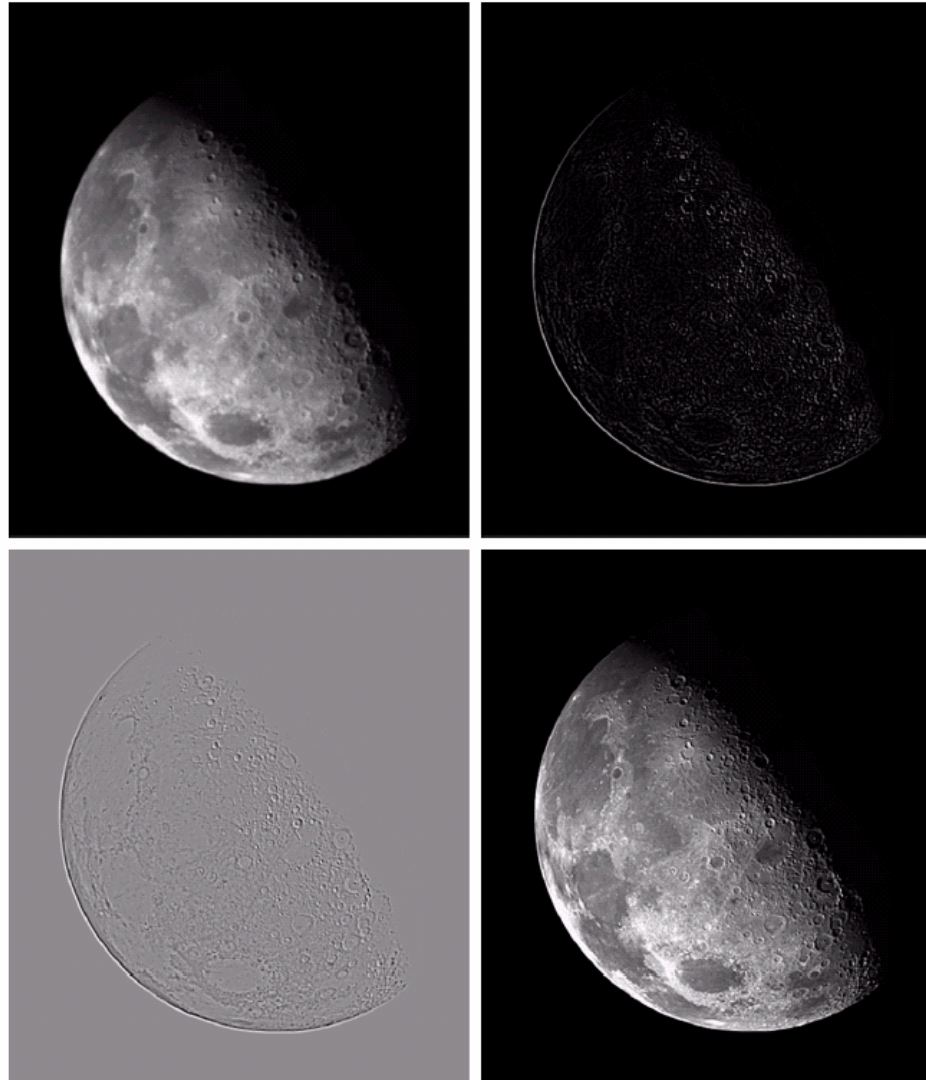


## Chapter 3 Image Enhancement in the Spatial Domain

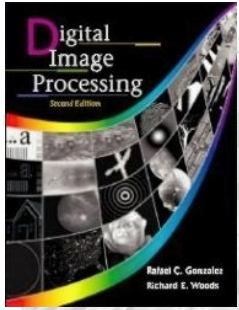
a b  
c d

**FIGURE 3.40**

(a) Image of the North Pole of the moon.  
(b) Laplacian-filtered image.  
(c) Laplacian image scaled for display purposes.  
(d) Image enhanced by using Eq. (3.7-5).  
(Original image courtesy of NASA.)



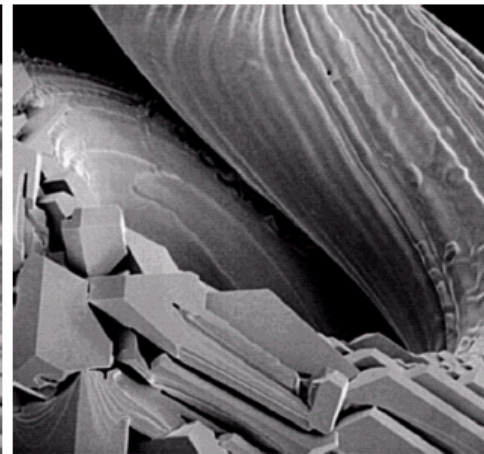
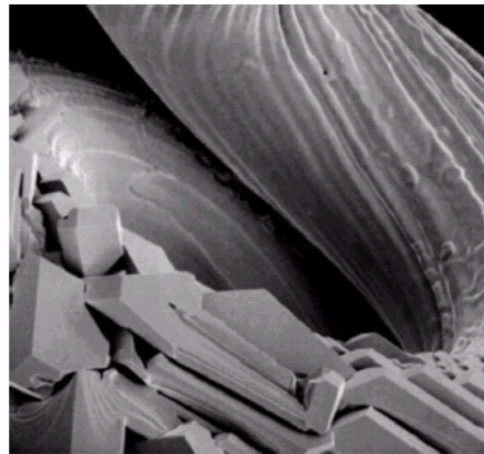
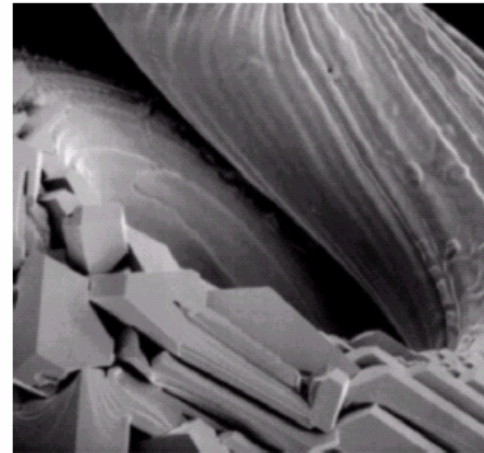




## Chapter 3 Image Enhancement in the Spatial Domain

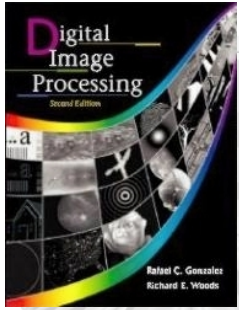
0	-1	0
-1	5	-1
0	-1	0

-1	-1	-1
-1	9	-1
-1	-1	-1



a b c  
d e

**FIGURE 3.41** (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

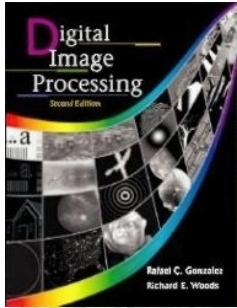


## Chapter 3 Image Enhancement in the Spatial Domain

0	-1	0	-1	-1	-1
-1	$A + 4$	-1	-1	$A + 8$	-1
0	-1	0	-1	-1	-1

a b

**FIGURE 3.42** The high-boost filtering technique can be implemented with either one of these masks, with  $A \geq 1$ .



## Chapter 3 Image Enhancement in the Spatial Domain

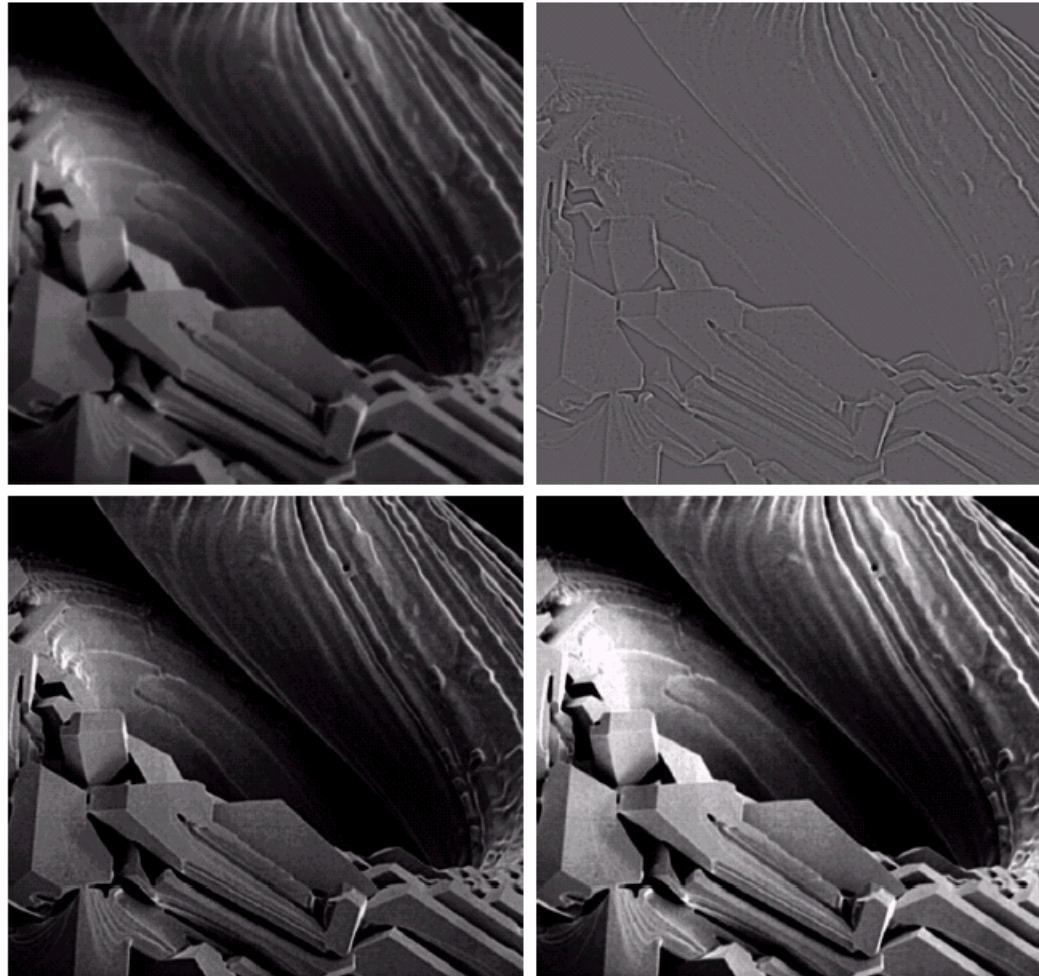
a b  
c d

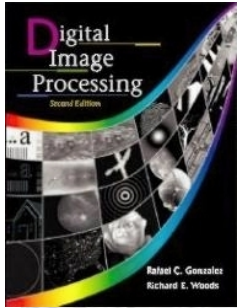
**FIGURE 3.43**

(a) Same as Fig. 3.41(c), but darker.

(b) Laplacian of (a) computed with the mask in Fig. 3.42(b) using  $A = 0$ .

(c) Laplacian enhanced image using the mask in Fig. 3.42(b) with  $A = 1$ . (d) Same as (c), but using  $A = 1.7$ .





# Chapter 3

## Image Enhancement in the Spatial Domain

a  
b c  
d e

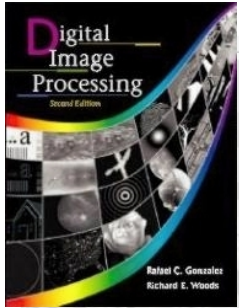
**FIGURE 3.44**  
A  $3 \times 3$  region of an image (the  $z$ 's are gray-level values) and masks used to compute the gradient at point labeled  $z_5$ . All masks coefficients sum to zero, as expected of a derivative operator.

$z_1$	$z_2$	$z_3$
$z_4$	$z_5$	$z_6$
$z_7$	$z_8$	$z_9$

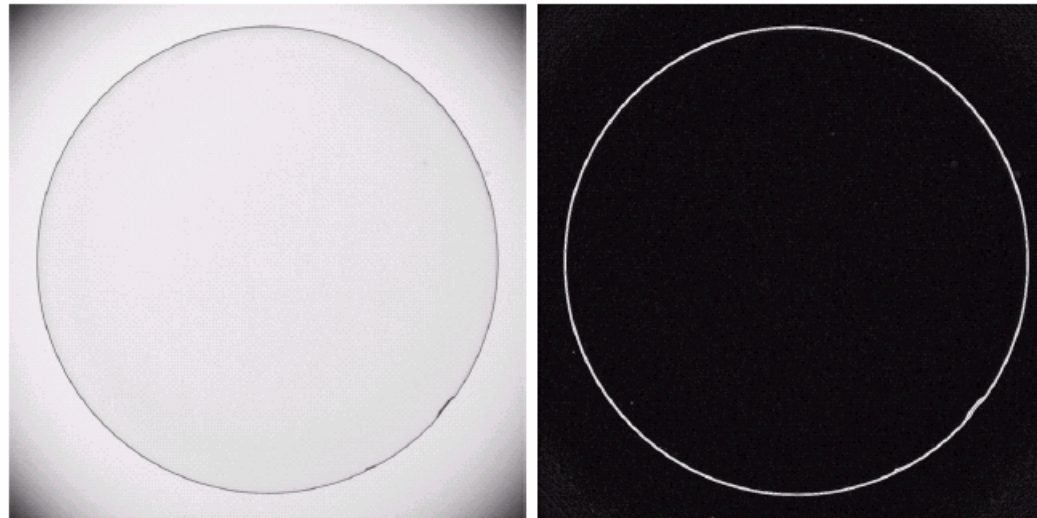
-1	0	0	-1
0	1	1	0

-1	-2	-1	-1	0	1
0	0	0	-2	0	2
1	2	1	-1	0	1





## Chapter 3 Image Enhancement in the Spatial Domain



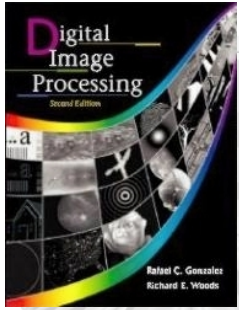
a b

**FIGURE 3.45**

Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock).

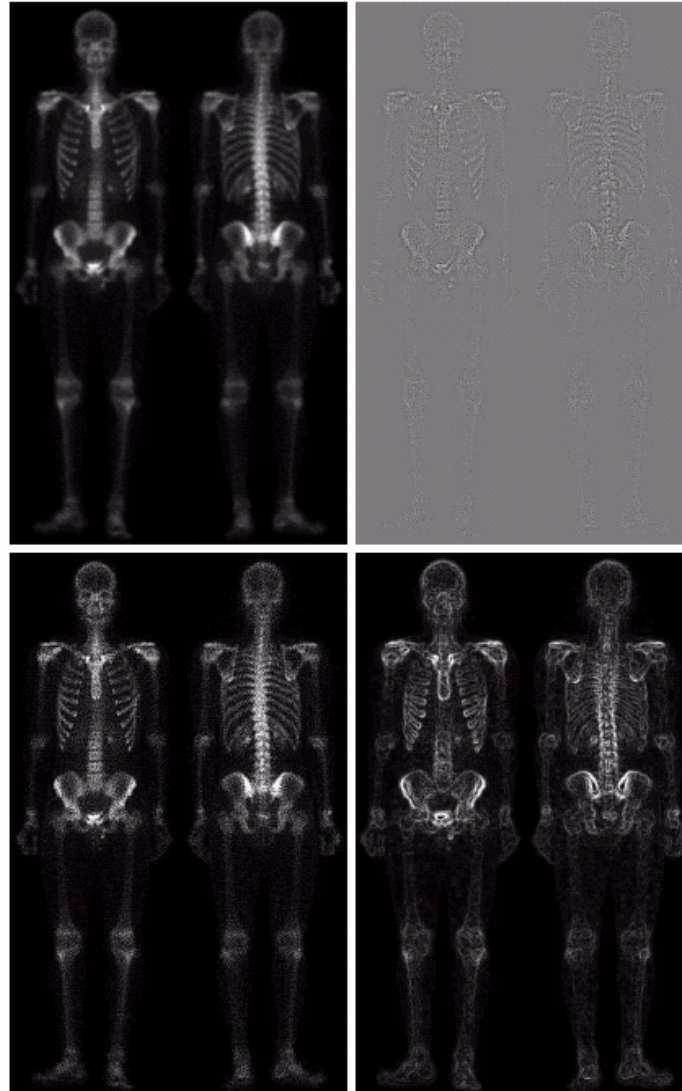
(b) Sobel gradient.

(Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)



## Chapter 3

# Image Enhancement in the Spatial Domain

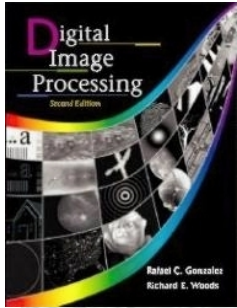


a b  
c d

**FIGURE 3.46**

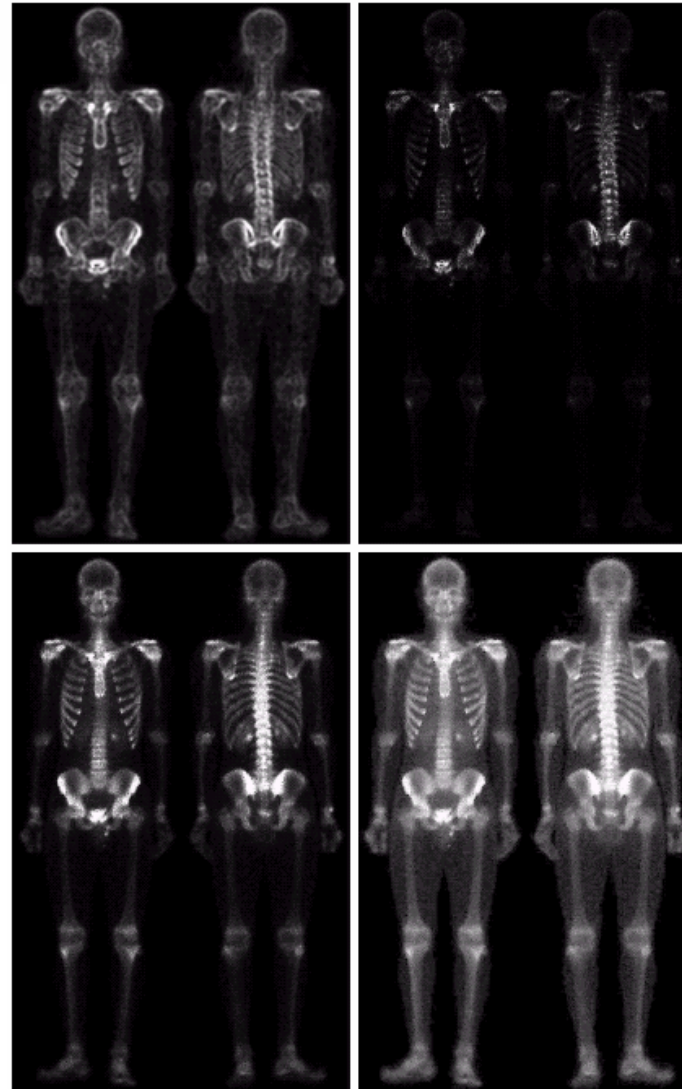
(a) Image of whole body bone scan. (b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel of (a).





## Chapter 3

# Image Enhancement in the Spatial Domain



e f  
g h

**FIGURE 3.46**

*(Continued)*

(e) Sobel image smoothed with a  $5 \times 5$  averaging filter. (f) Mask image formed by the product of (c) and (e).

(g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)